Objectives

- To discuss the concept of the center of gravity, center of mass, and centroids (centers of area).
- To show how to determine the location of the center of gravity and centroid for a system of particles and a body of arbitrary shape.

Center of Gravity

The center of gravity G is a point which locates the resultant weight of a system of particles.

The weights of the particles is considered to be a parallel force system. The system of weights can be replaced by a single weight acting at the Center of Gravity.

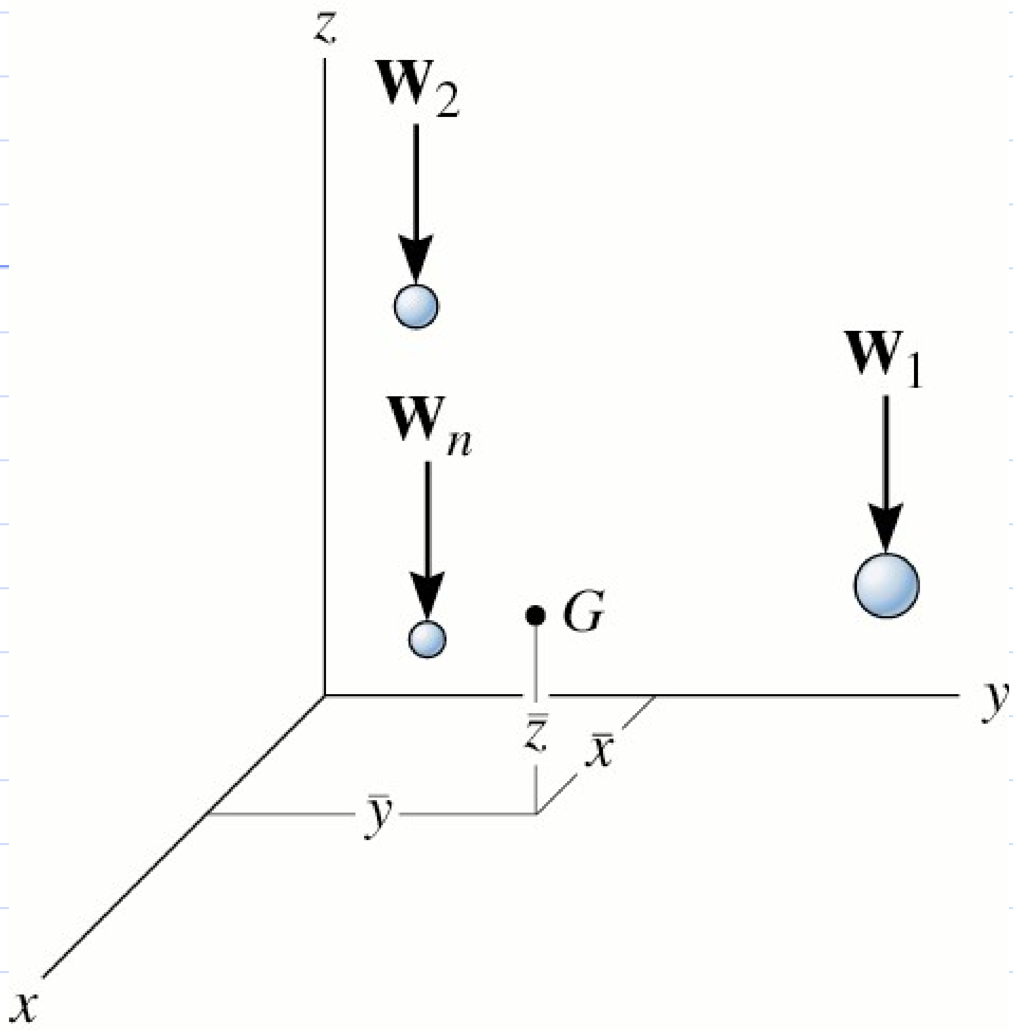


Figure 09.01(a)

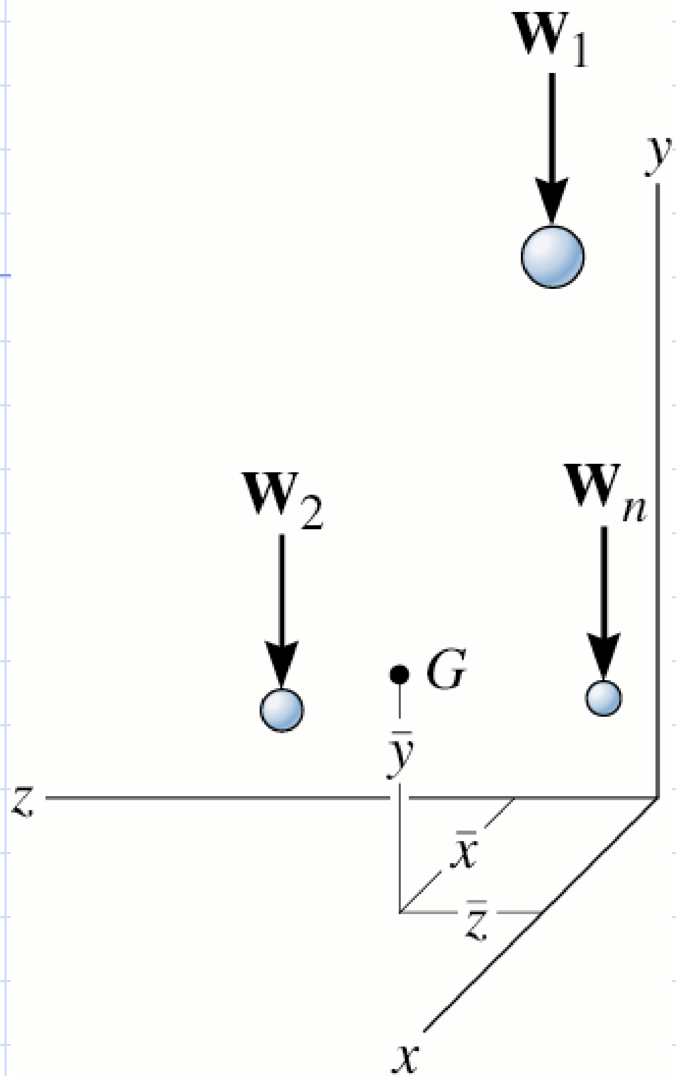


Figure 09.01(b)

$$W_R = \sum_{i=1}^{n} W_i$$

Total Weight

x location:

$$\bar{x}_R W_R = \tilde{x}_1 W_1 + \tilde{x}_2 W_2 + \tilde{x}_3 W_3 + \quad \tilde{x}_n W_n$$

y location:

$$\bar{y}_R W_R = \tilde{y}_1 W_1 + \tilde{y}_2 W_2 + \tilde{y}_3 W_3 + \quad \tilde{y}_n W_n$$

z location:

$$\bar{z}_R W_R = \tilde{z}_1 W_1 + \tilde{z}_2 W_2 + \tilde{z}_3 W_3 + \quad \tilde{z}_n W_n$$

$$\bar{x} = \frac{\sum_{i=1}^{n} \tilde{x}_i W_i}{\sum_{i=1}^{n} W_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} \tilde{y}_i W_i}{\sum_{i=1}^{n} W_i} \qquad \bar{z} = \frac{\sum_{i=1}^{n} \tilde{z}_i W_i}{\sum_{i=1}^{n} W_i}$$

$\bar{x}, \bar{y}, \bar{z}$ coordinates of the center of gravit

$\tilde{x}_i, \tilde{y}_i, \tilde{z}_i$ coordinates of the i^{th} particle

W_i weight of the i^{th} particle

Center of Mass

$$\bar{x} = \frac{\sum_{i=1}^{n} \tilde{x}_i m_i}{\sum_{i=1}^{n} m_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} \tilde{y}_i m_i}{\sum_{i=1}^{n} m_i} \qquad \bar{z} = \frac{\sum_{i=1}^{n} \tilde{z}_i m_i}{\sum_{i=1}^{n} m_i}$$

$\bar{x}, \bar{y}, \bar{z}$ coordinates of the center of mas

$\tilde{x}_i, \tilde{y}_i, \tilde{z}_i$ coordinates of the i^{th} particle

W_i mass of the i^{th} particle

Center of Gravity and Centroid for a Body

Consider a body to be a system of an infinite number of particles

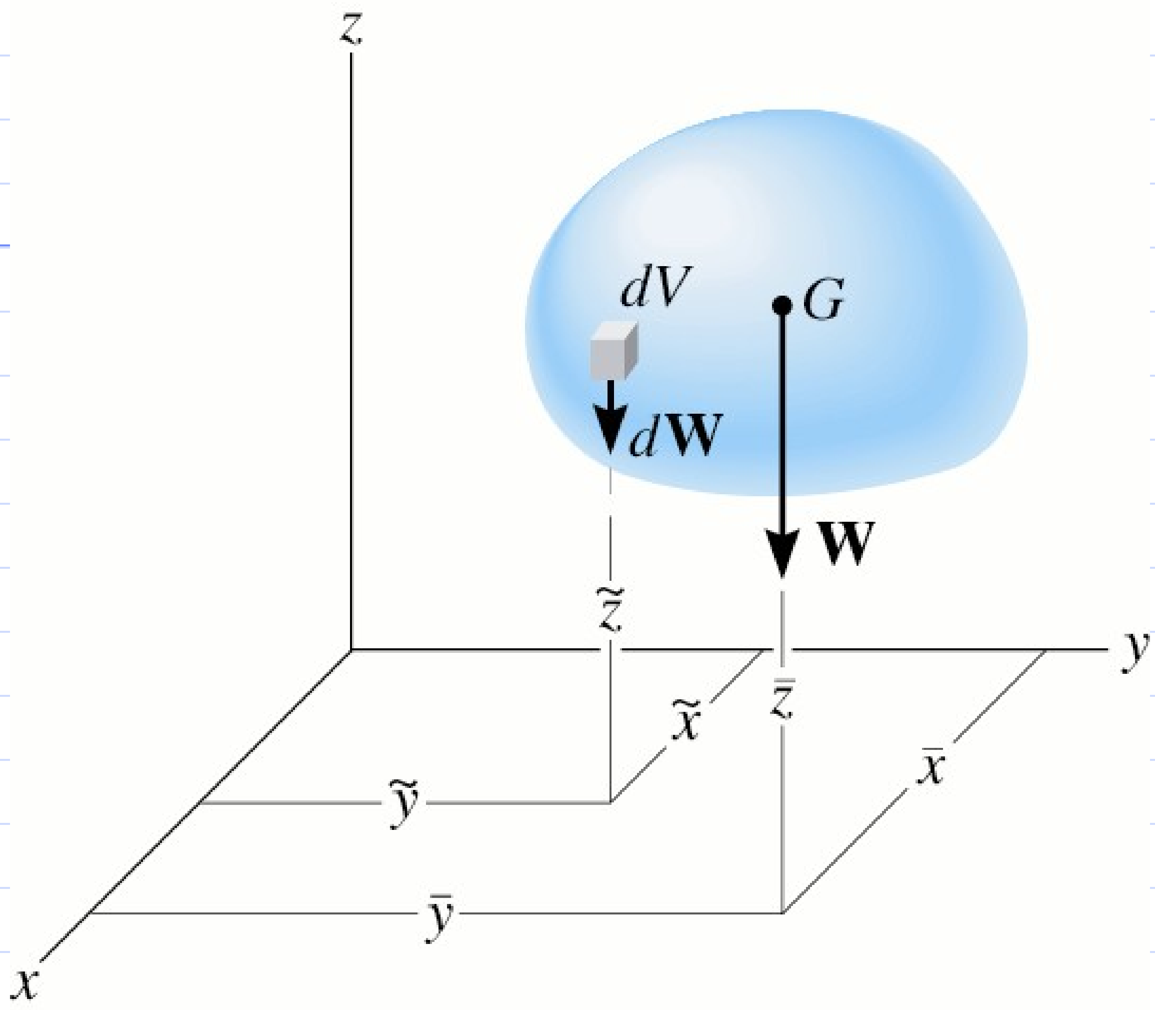


Figure 09.02

$$\bar{x} = \frac{\sum_{i=1}^{\infty} \tilde{x}_i W_i}{\sum_{i=1}^{\infty} W_i} \qquad \bar{y} = \frac{\sum_{i=1}^{\infty} \tilde{y}_i W_i}{\sum_{i=1}^{\infty} W_i} \qquad \bar{z} = \frac{\sum_{i=1}^{\infty} \tilde{z}_i W_i}{\sum_{i=1}^{\infty} W_i}$$

$\bar{x}, \bar{y}, \bar{z}$ coordinates of the center of gravit

$\tilde{x}_i, \tilde{y}_i, \tilde{z}_i$ coordinates of the i^{th} particle

W_i weight of the i^{th} particle

$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\, dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

γ - specific weight of the body

(The weight per unit volume)

$$dW = \gamma\, dV$$

Center of Gravity of a Body

$$\bar{x} = \frac{\int_V \tilde{x}\gamma dV}{\int_V \gamma dV} \qquad \bar{y} = \frac{\int_V \tilde{y}\gamma dV}{\int_V \gamma dV} \qquad \bar{z} = \frac{\int_V \tilde{z}\gamma dV}{\int_V \gamma dV}$$

CENTROID

The centroid C is a point which defines the geometric center of an object. Its location can be determined by formulas similar to those used for center of gravity or center of mass.

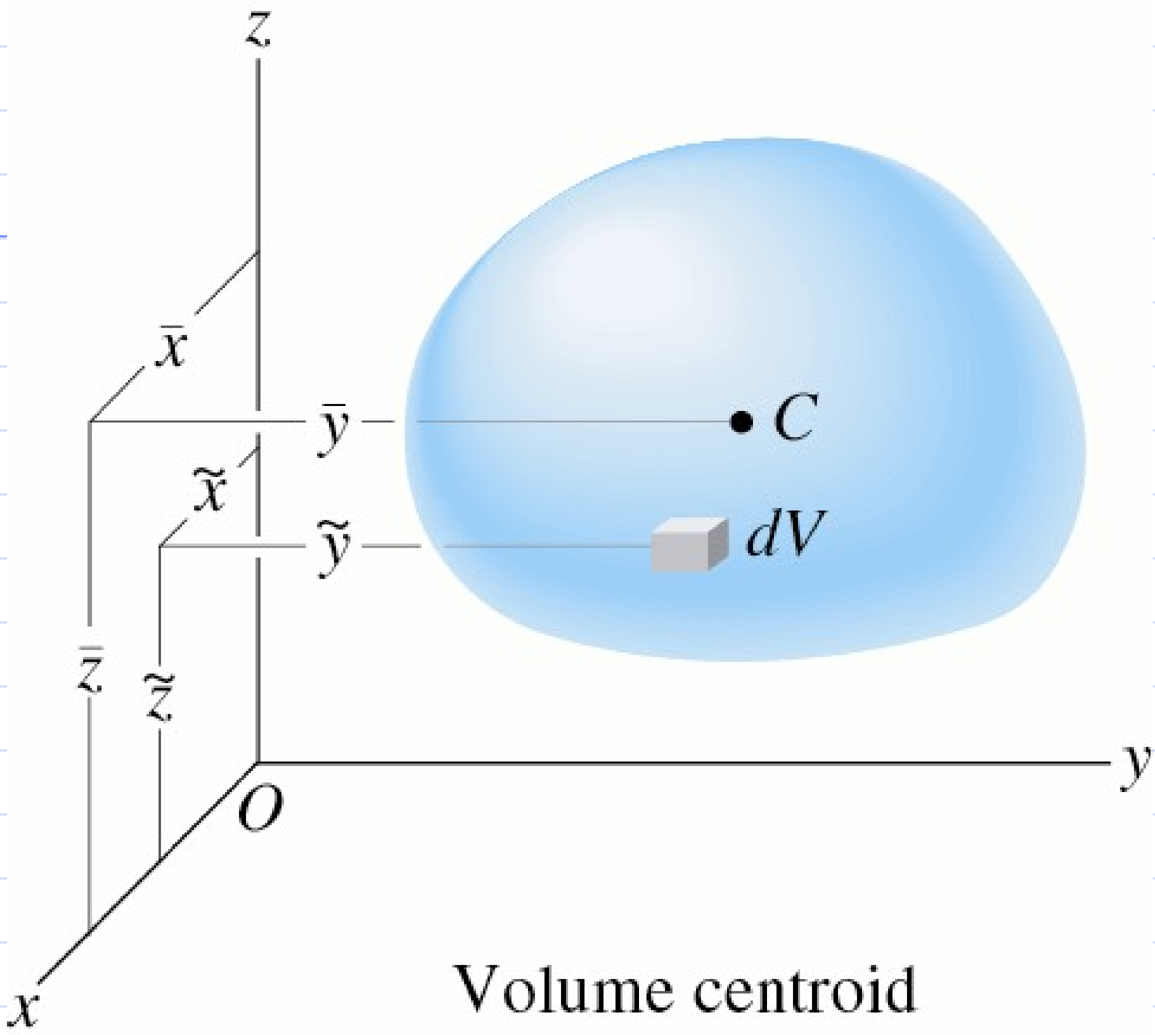
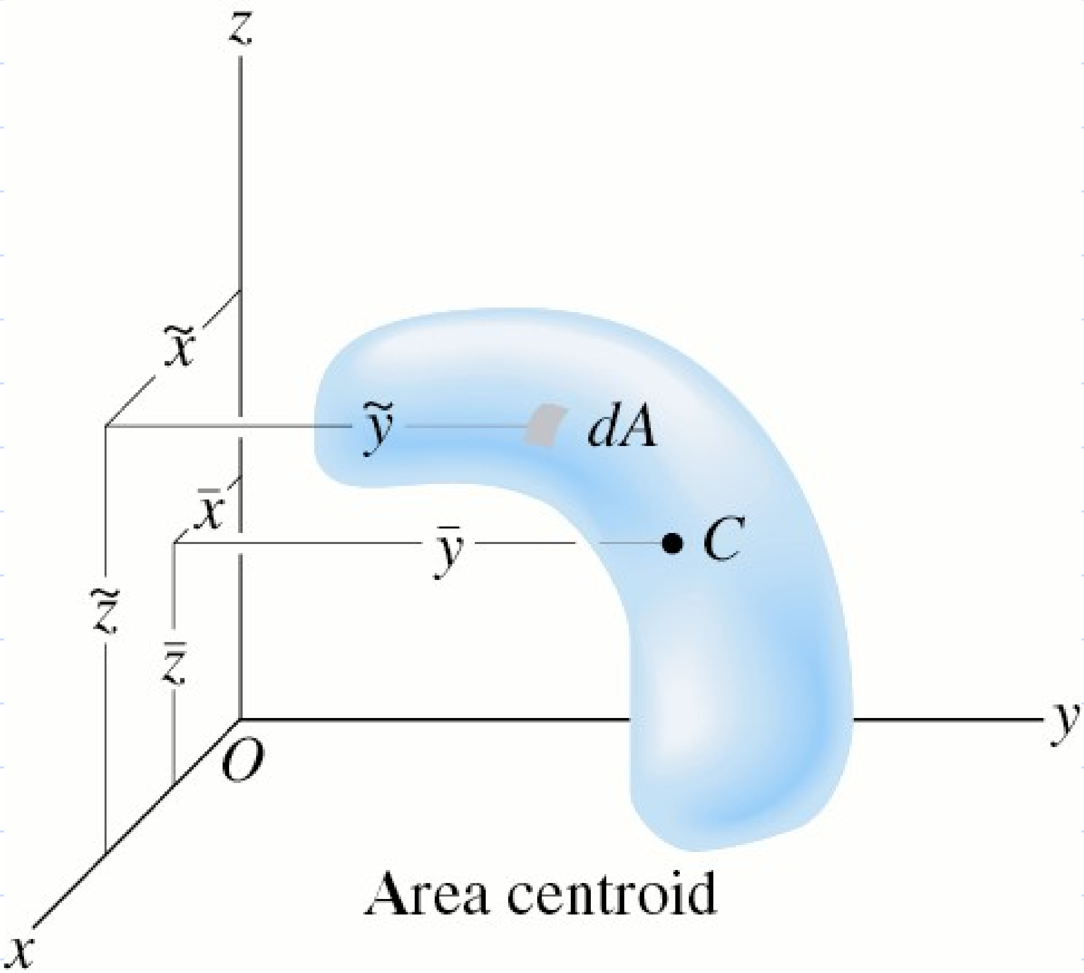

z
$\bar{x}$
$\bar{y}$
C
$\tilde{x}$
$\tilde{y}$
dV
$\bar{z}$
$\tilde{z}$
O
y
x
Volume centroid

Figure 09.03

Centroid of a Volume

$$\bar{x} = \frac{\int_V \tilde{x}\, dV}{\int_V dV} \qquad \bar{y} = \frac{\int_V \tilde{y}\, dV}{\int_V dV} \qquad \bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV}$$



Area centroid

Figure 09.04

Centroid of an Area

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} \qquad \bar{z} = \frac{\int_A \tilde{z}\, dA}{\int_A dA}$$

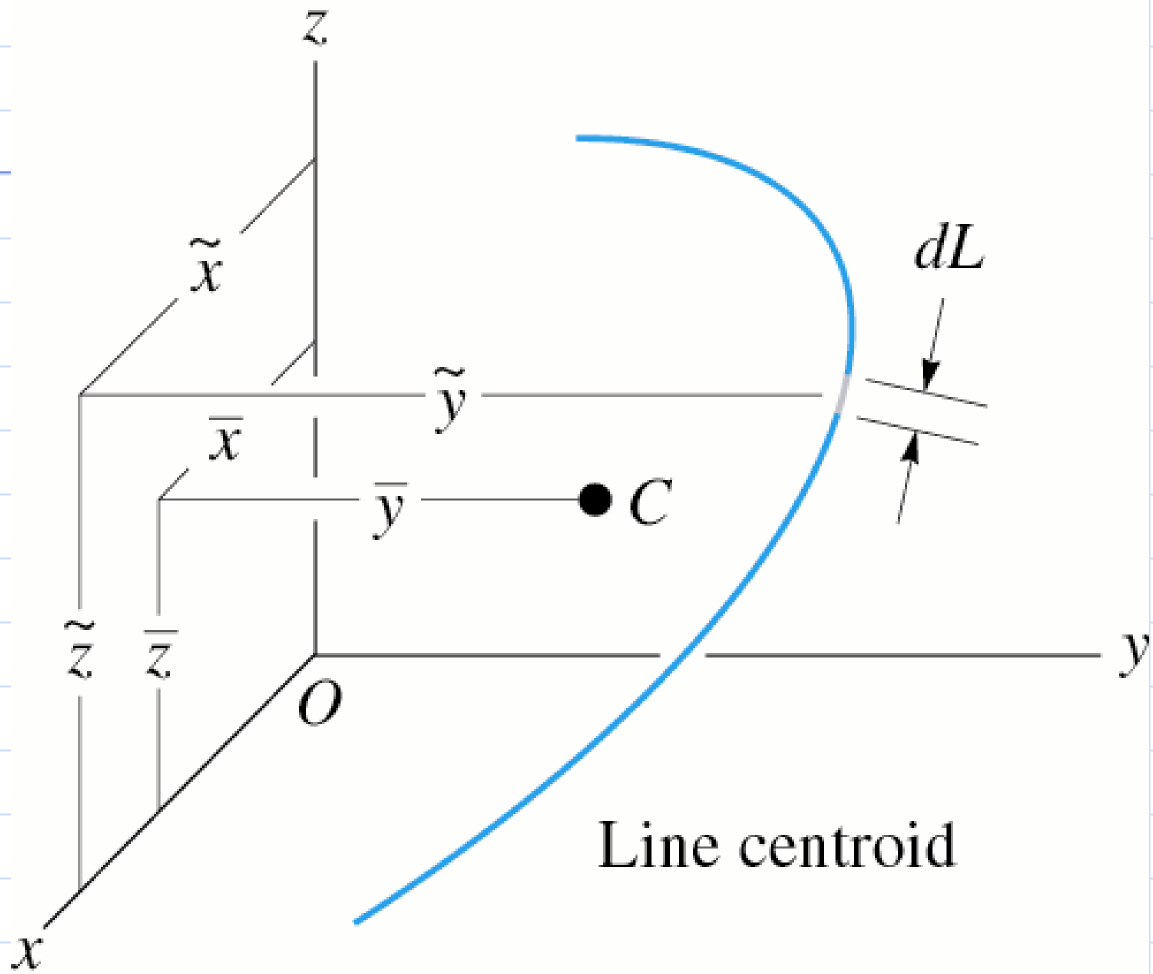


Figure 09.05

Centroid of a Line

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} \qquad \bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} \qquad \bar{z} = \frac{\int_L \tilde{z}\,dL}{\int_L dL}$$

Figure 09.05.01(C)

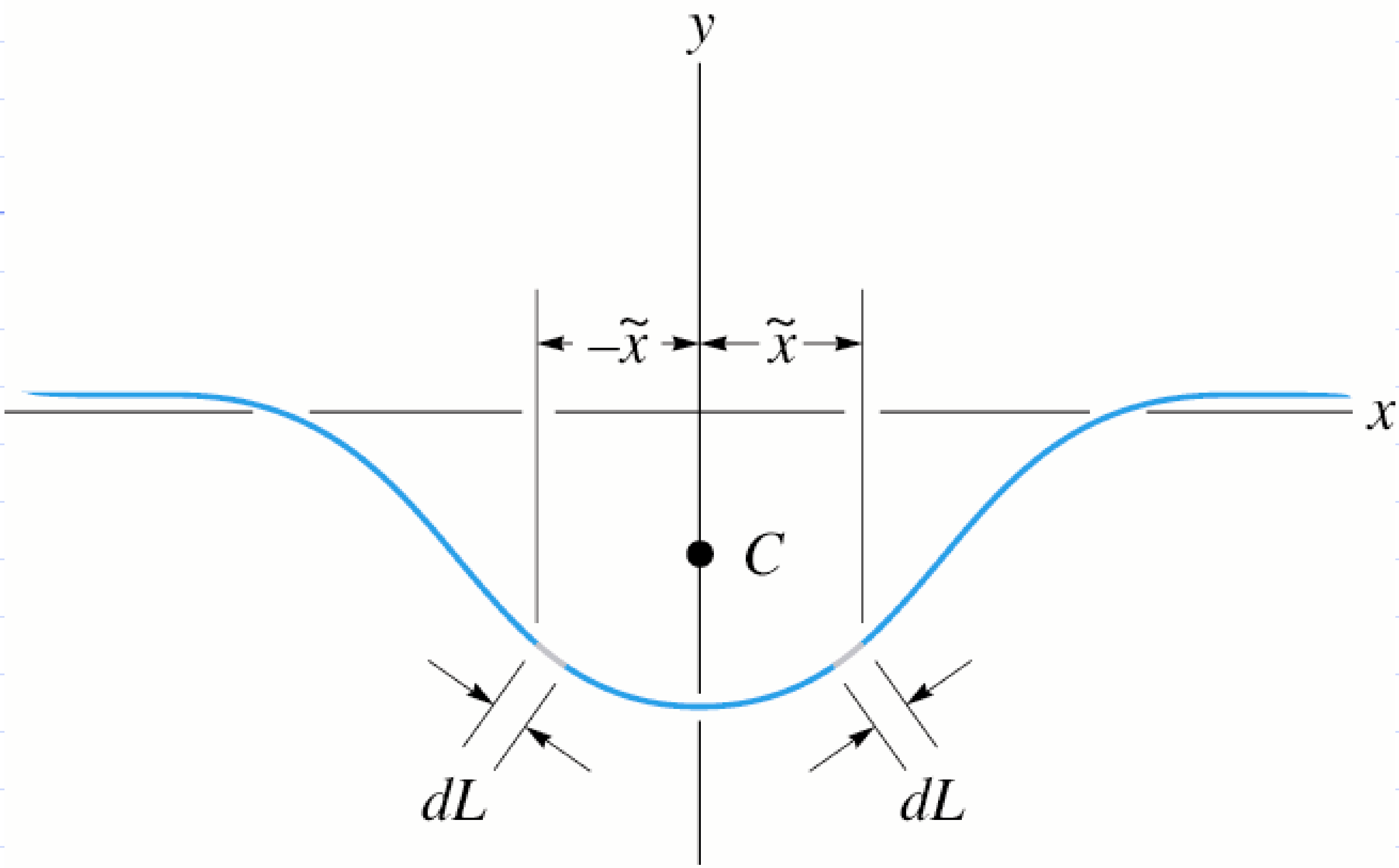


Figure 09.06

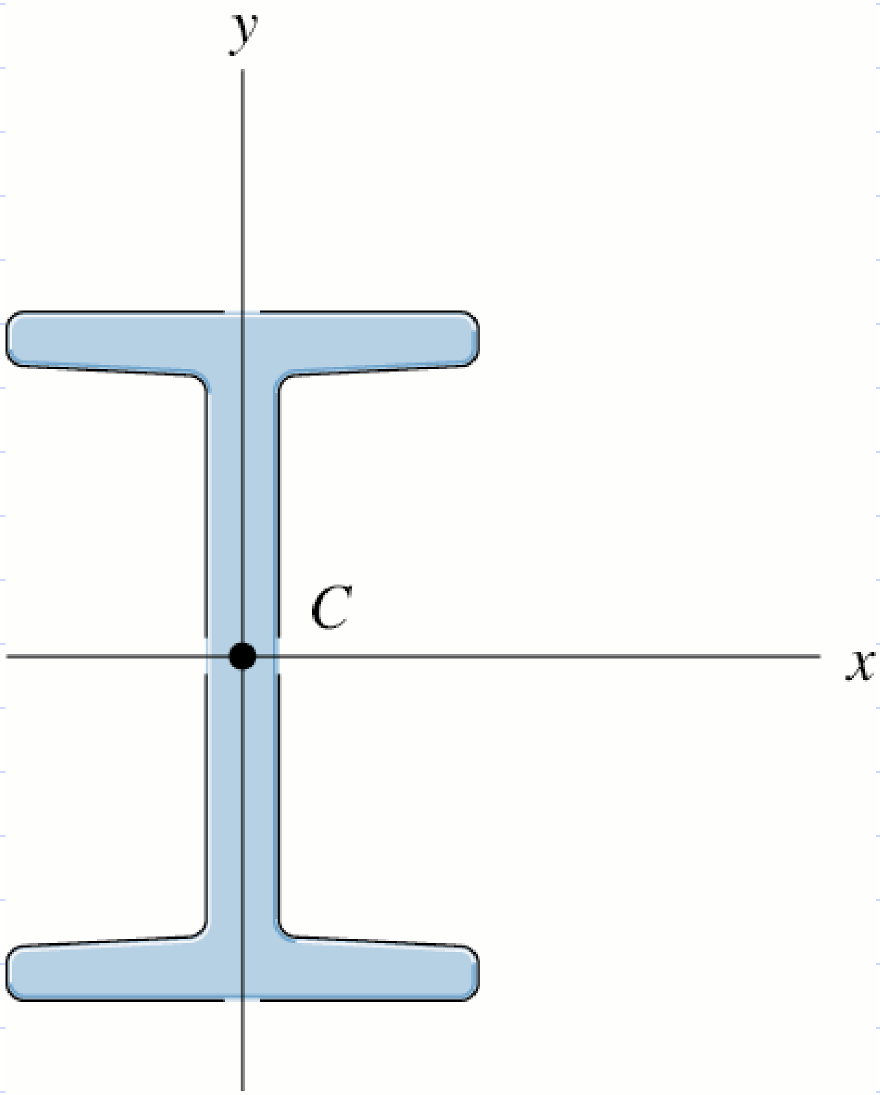


Figure 09.07

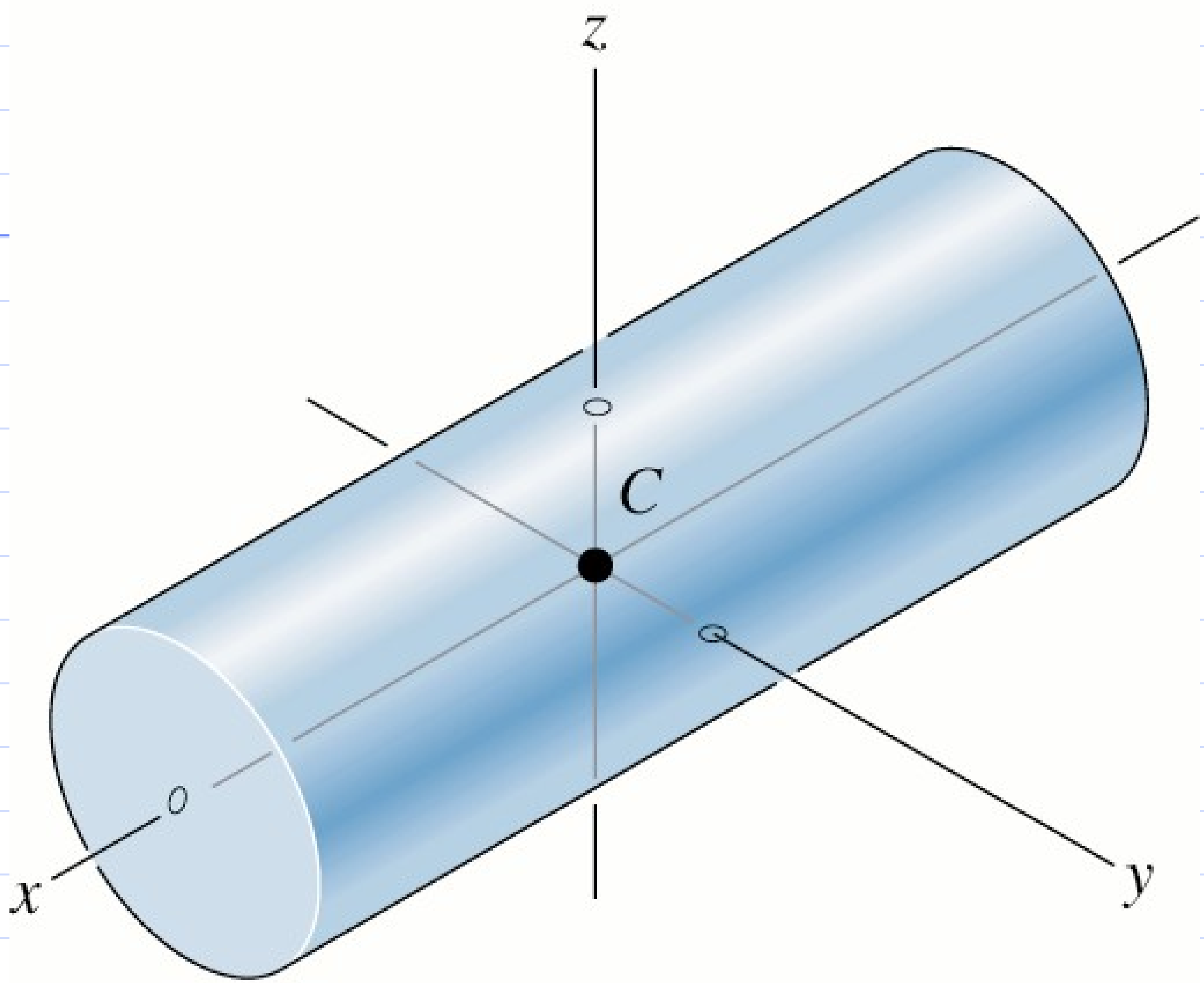


Figure 09.08

PROBLEM

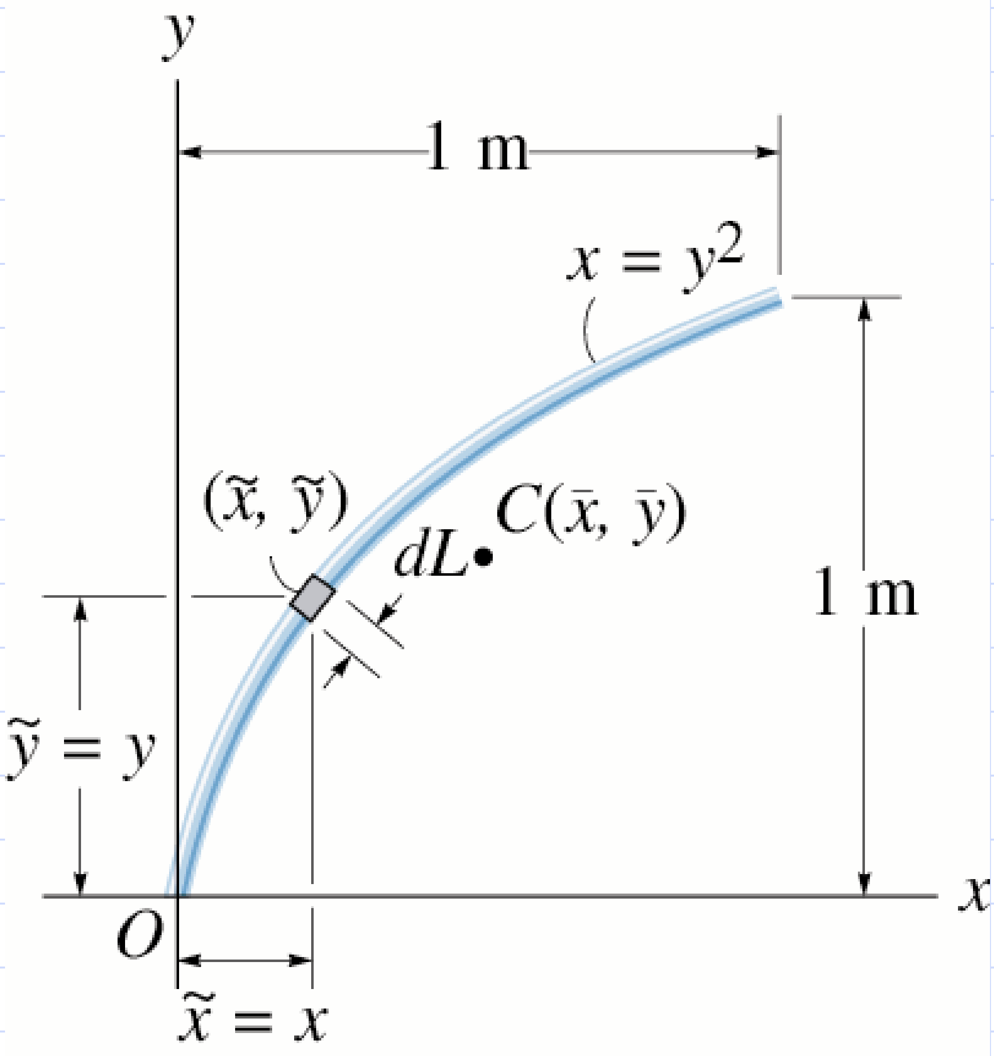


Figure 09.09

$$dL = \sqrt{(dx)^2 + (dy)^2}$$

$$dL = \left(\sqrt{\left(\frac{dx}{dy} \right)^2 + 1} \right) dy$$

$$x = y^2$$

$$\frac{dx}{dy} = 2y$$

$$dL = \left(\sqrt{(2y)^2 + 1} \right) dy$$

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{\int_0^1 x\left(\sqrt{(2y)^2+1}\right)dy}{\int_0^1 \left(\sqrt{(2y)^2+1}\right)dy}$$

$$x = y^2$$

$$\bar{x} = \frac{\int_0^1 y^2\left(\sqrt{4y^2+1}\right)dy}{\int_0^1 \left(\sqrt{4y^2+1}\right)dy} = \frac{0.6063}{1.479} = 0.410\ \text{m}$$

$$\bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL} = \frac{\int_0^1 y\left(\sqrt{(2y)^2 + 1}\right) dy}{\int_0^1 \left(\sqrt{(2y)^2 + 1}\right) dy}$$

$$\bar{y} = \frac{\int_0^1 y\left(\sqrt{4y^2 + 1}\right) dy}{\int_0^1 \left(\sqrt{4y^2 + 1}\right) dy} = \frac{0.8484}{1.479} = 0.574\,m$$

PROBLEM

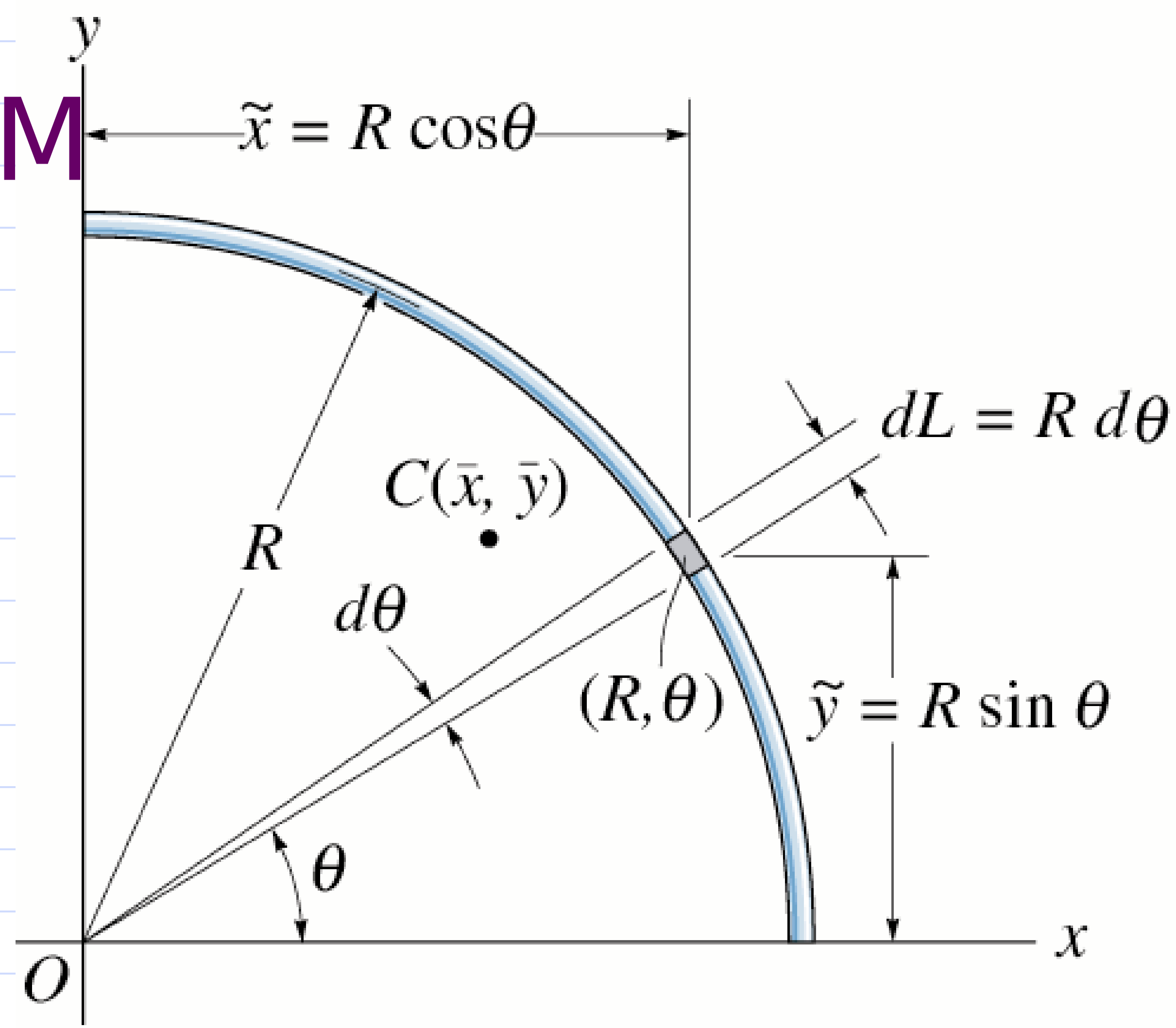
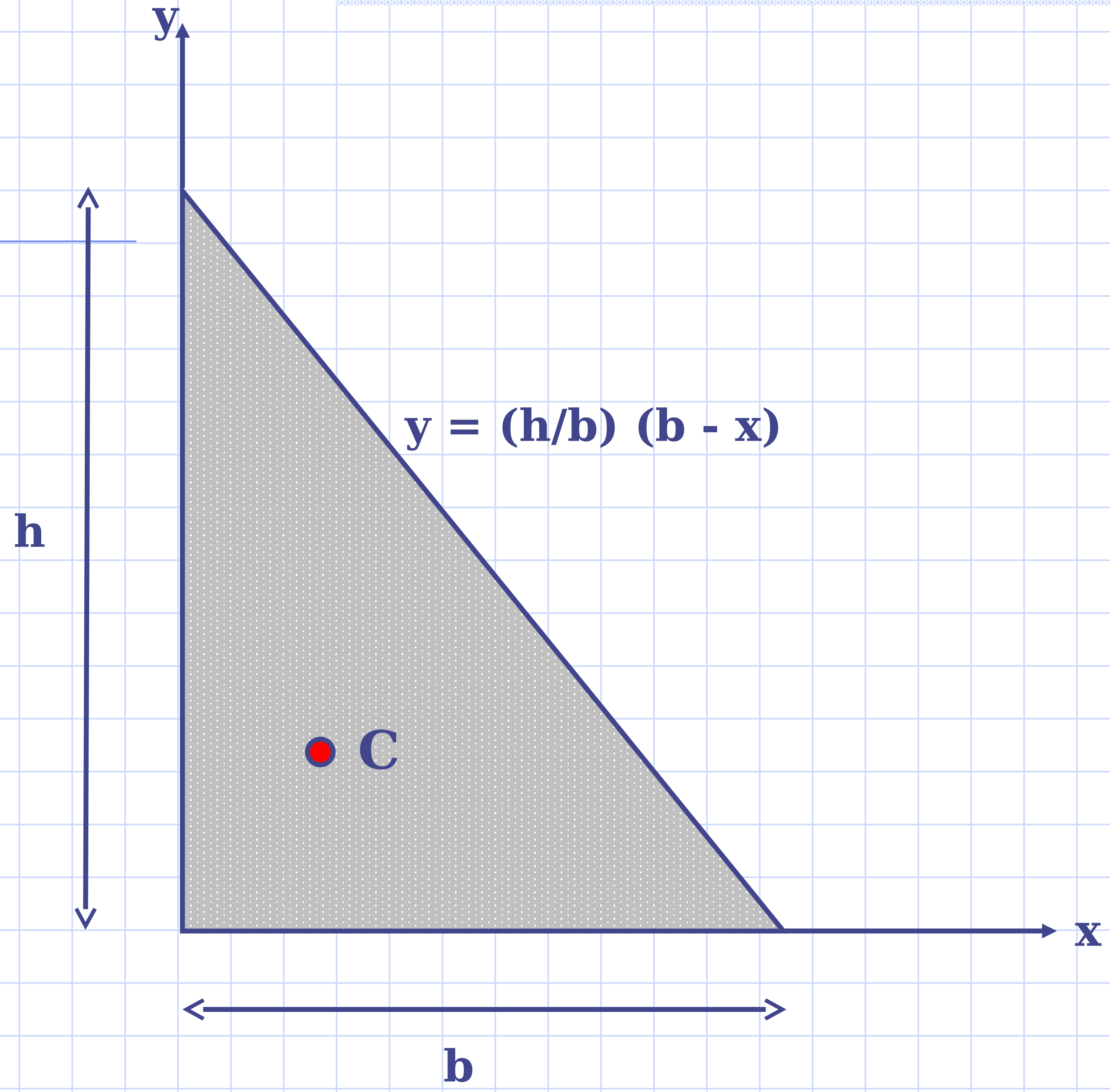


Figure 09.10

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{\int_0^{\frac{\pi}{2}} (R\cos\theta)\, R\,d\theta}{\int_0^{\frac{\pi}{2}} R\,d\theta} = \frac{R^2 \int_0^{\frac{\pi}{2}} \cos\theta\, d\theta}{R \int_0^{\frac{\pi}{2}} d\theta} = \frac{2R}{\pi}$$

$$\bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} = \frac{\int_0^{\frac{\pi}{2}} (R\sin\theta)\, R\,d\theta}{\int_0^{\frac{\pi}{2}} R\,d\theta} = \frac{R^2 \int_0^{\frac{\pi}{2}} \sin\theta\, d\theta}{R \int_0^{\frac{\pi}{2}} d\theta} = \frac{2R}{\pi}$$


y
y = (h/b) (b - x)
h
C
x
b

Strip Method

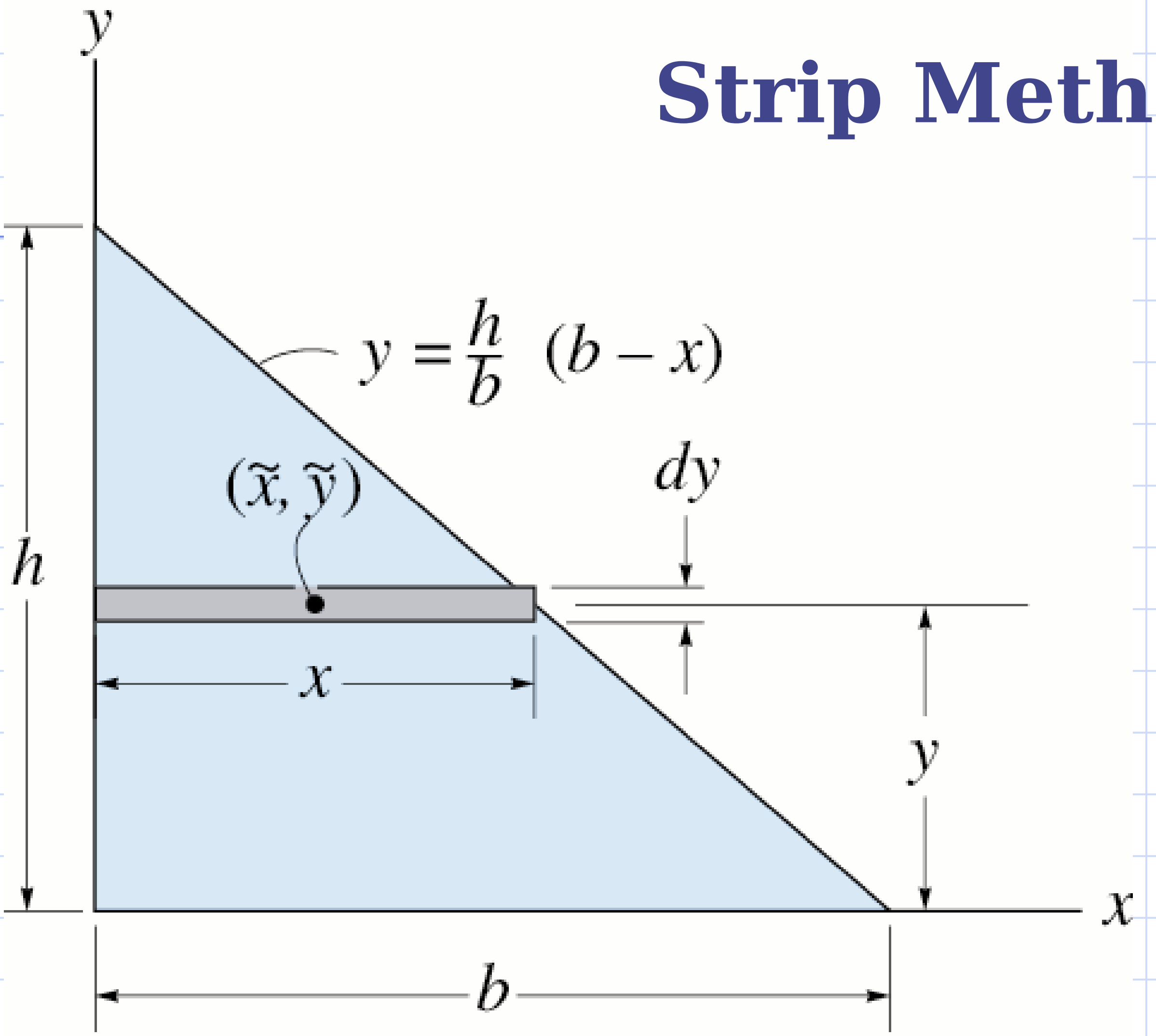


Figure 09.11

$$dA = x\,dy$$

$$dA = \frac{b}{h}(h - y)\,dy$$

$$\tilde{x} = \frac{1}{2}\left(\frac{b}{h}(h - y)\right)$$

$$\tilde{y} = y$$

$$\bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} = \frac{\int_0^h y\left(\frac{b}{h}(h-y)\right)dy}{\int_0^h \left(\frac{b}{h}(h-y)\right)dy}$$

$$\bar{y} = \frac{\frac{1}{6}bh^2}{\frac{1}{2}bh} = \frac{h}{3}$$

$$\bar{x} = \frac{\int_A \tilde{x} \, dA}{\int_A dA} = \frac{\int_0^h \frac{1}{2}\left(\frac{b}{h}(h-y)\right)\left(\frac{b}{h}(h-y)\right) dy}{\int_0^h \left(\frac{b}{h}(h-y)\right) dy}$$

$$\bar{x} = \frac{\frac{1}{6} \, b^2 \, h}{\frac{1}{2} \, b \, h} = \frac{b}{3}$$

PROBLEM

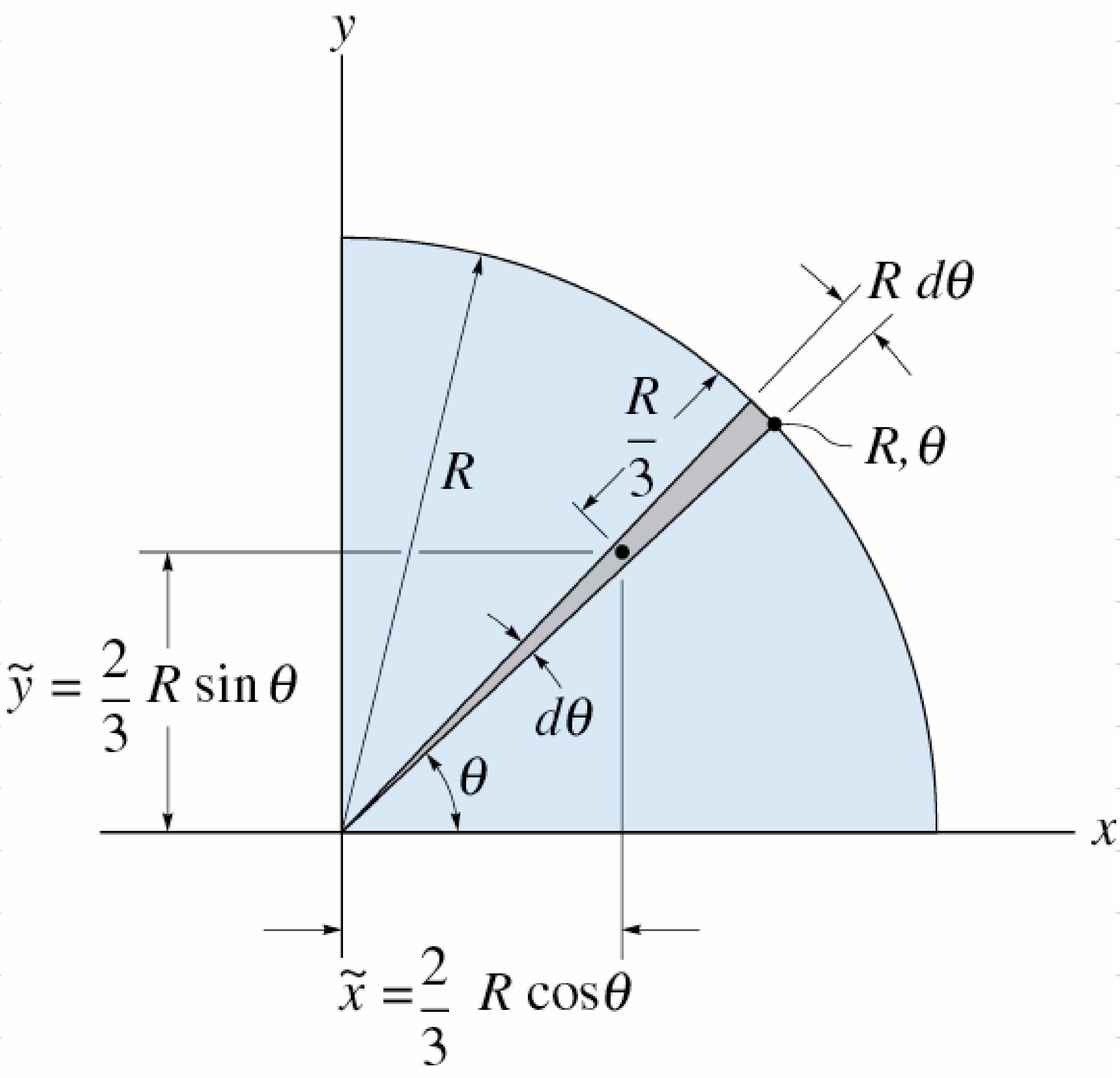


Figure 09.12(a)

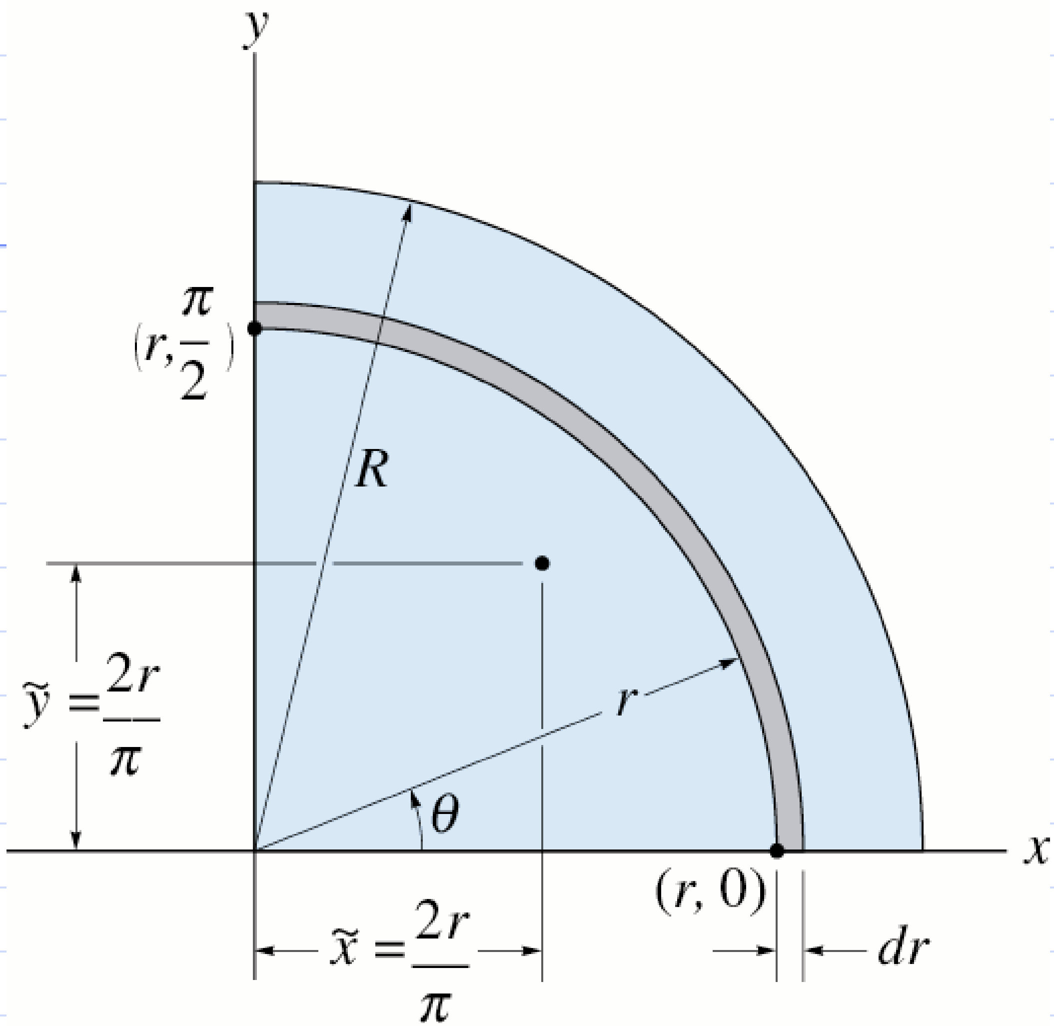


Figure 09.12(b)

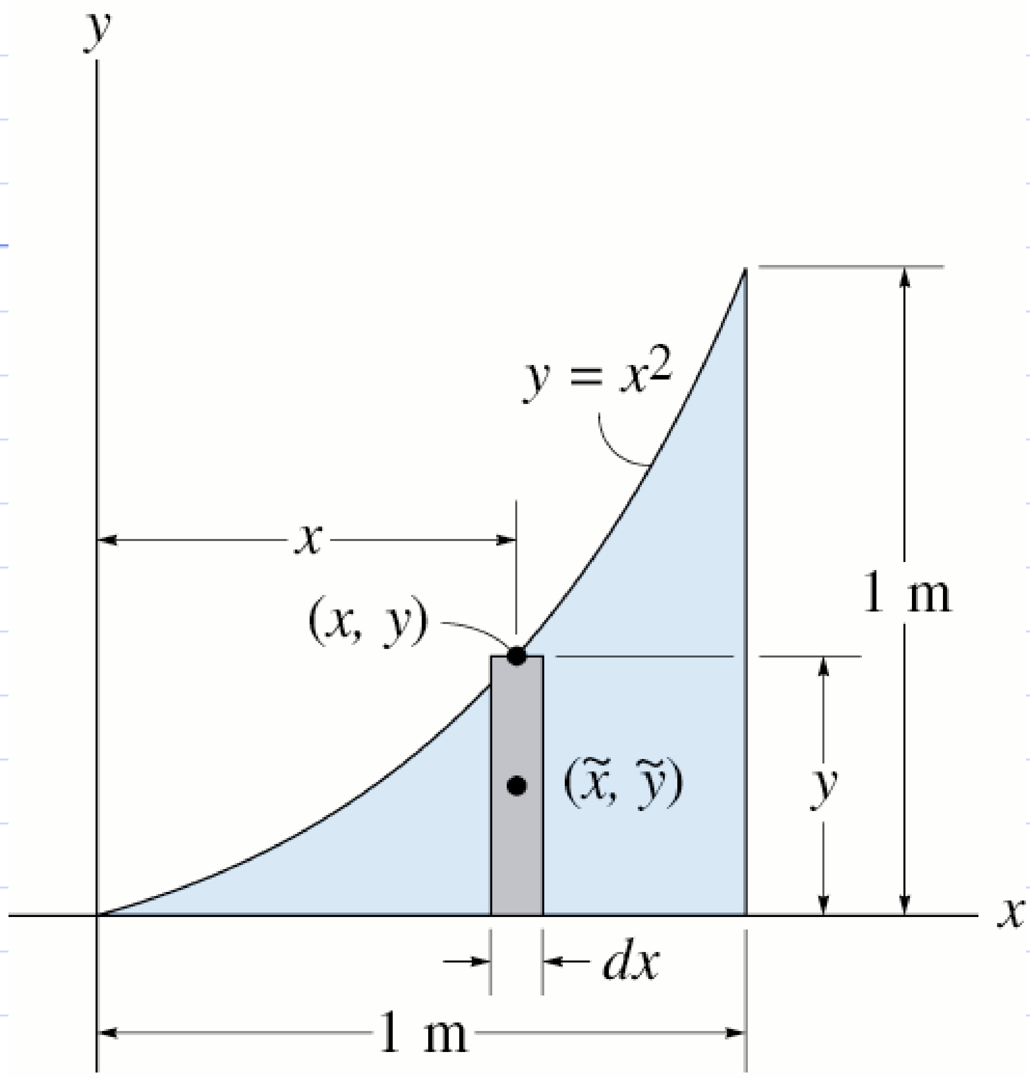


Figure 09.13(a)

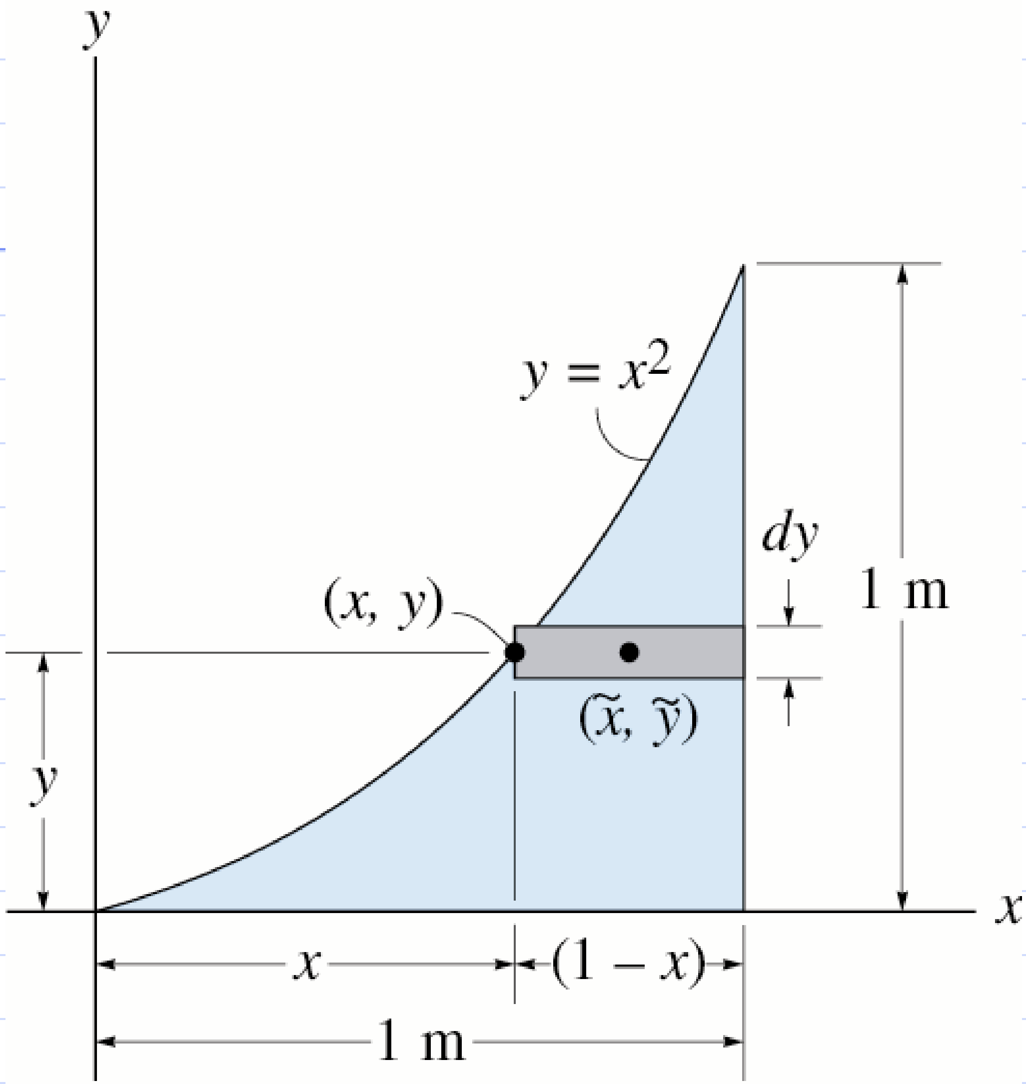


Figure 09.13(b)

Composite Bodies

Figure 09.16.01(C)

Composite Bodies

If a body is made up of several simpler bodies then a special technique can be used.

Procedure

- Divide body into several subparts.
- If the body has a hole or cutout, treat that as negative area.
- Centroid will lie on line of symmetry.
- Create Table and calculate centroid.

$$\bar{x} = \frac{\sum_{i=1}^{n} \tilde{x}_i A_i}{\sum_{i=1}^{n} A_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} \tilde{y}_i A_i}{\sum_{i=1}^{n} A_i} \qquad \bar{z} = \frac{\sum_{i=1}^{n} \tilde{z}_i A_i}{\sum_{i=1}^{n} A_i}$$

$\bar{x}, \bar{y}, \bar{z}$ coordinates of the center of gravit

$\tilde{x}_i, \tilde{y}_i, \tilde{z}_i$ coordinates of the i^{th} particle

W_i weight of the i^{th} particle

Body	Area	x_c	y_c	$x_c A$	$y_c A$

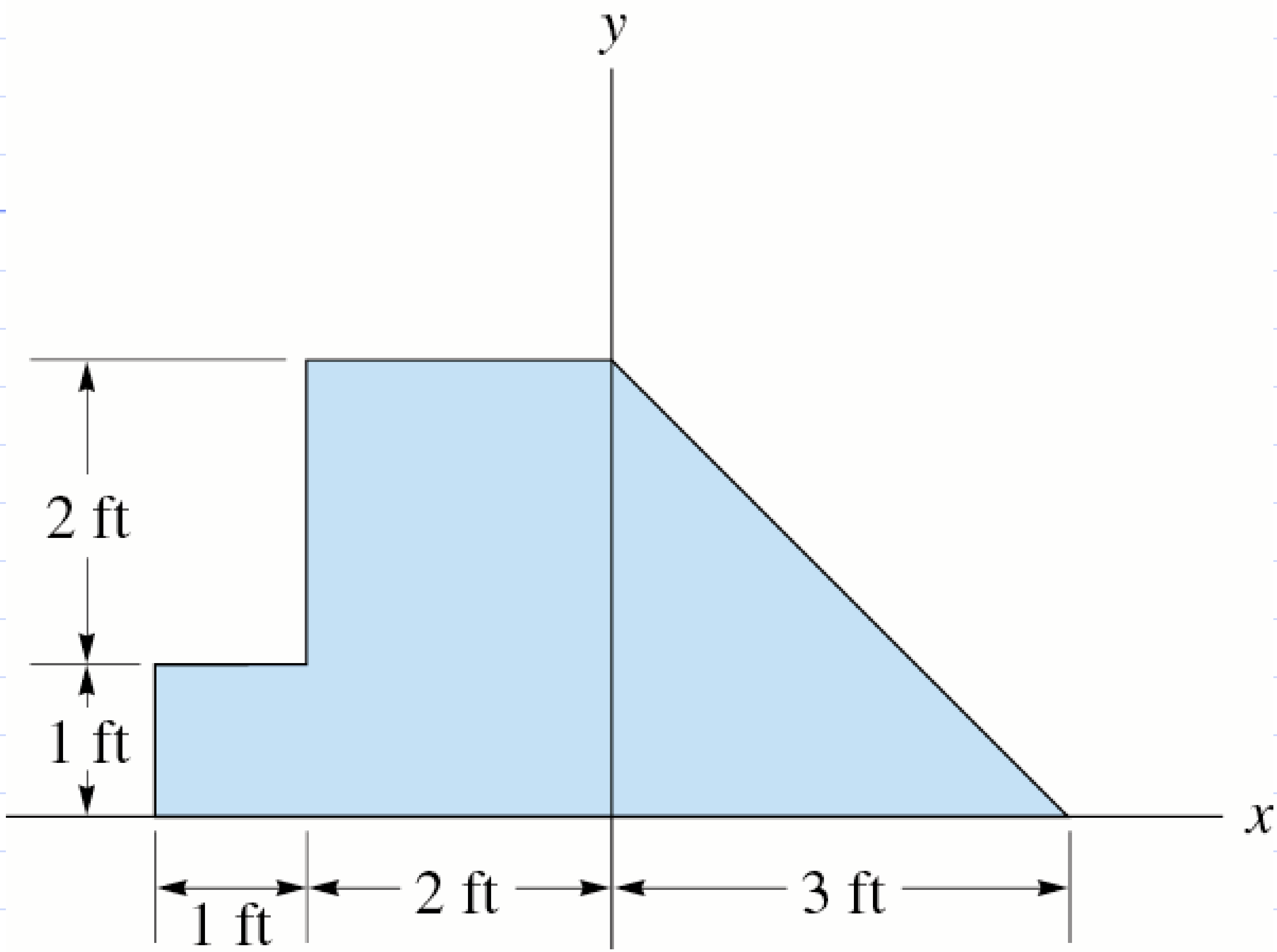


Figure 09.18(a)

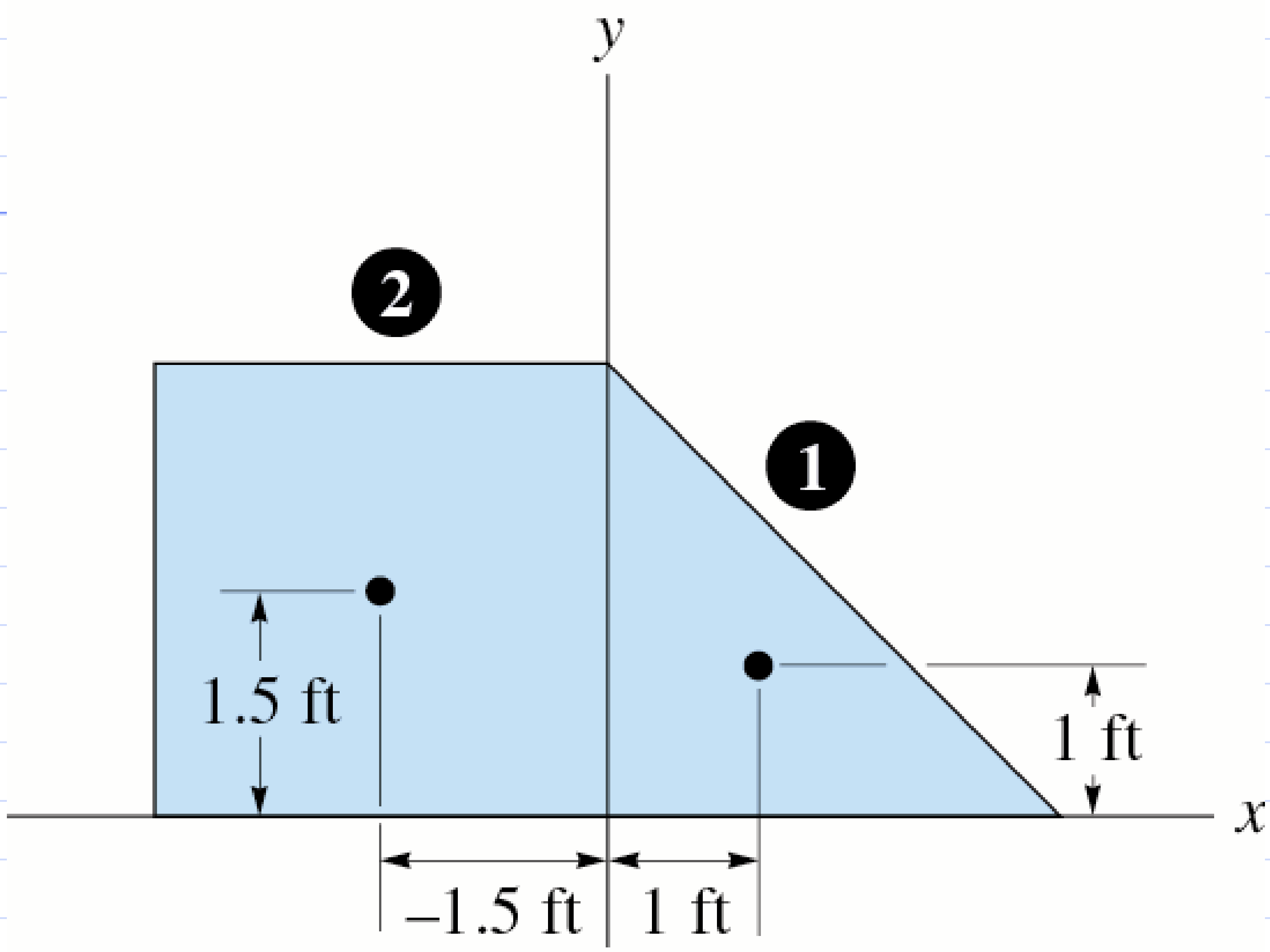


Figure 09.18(b1)

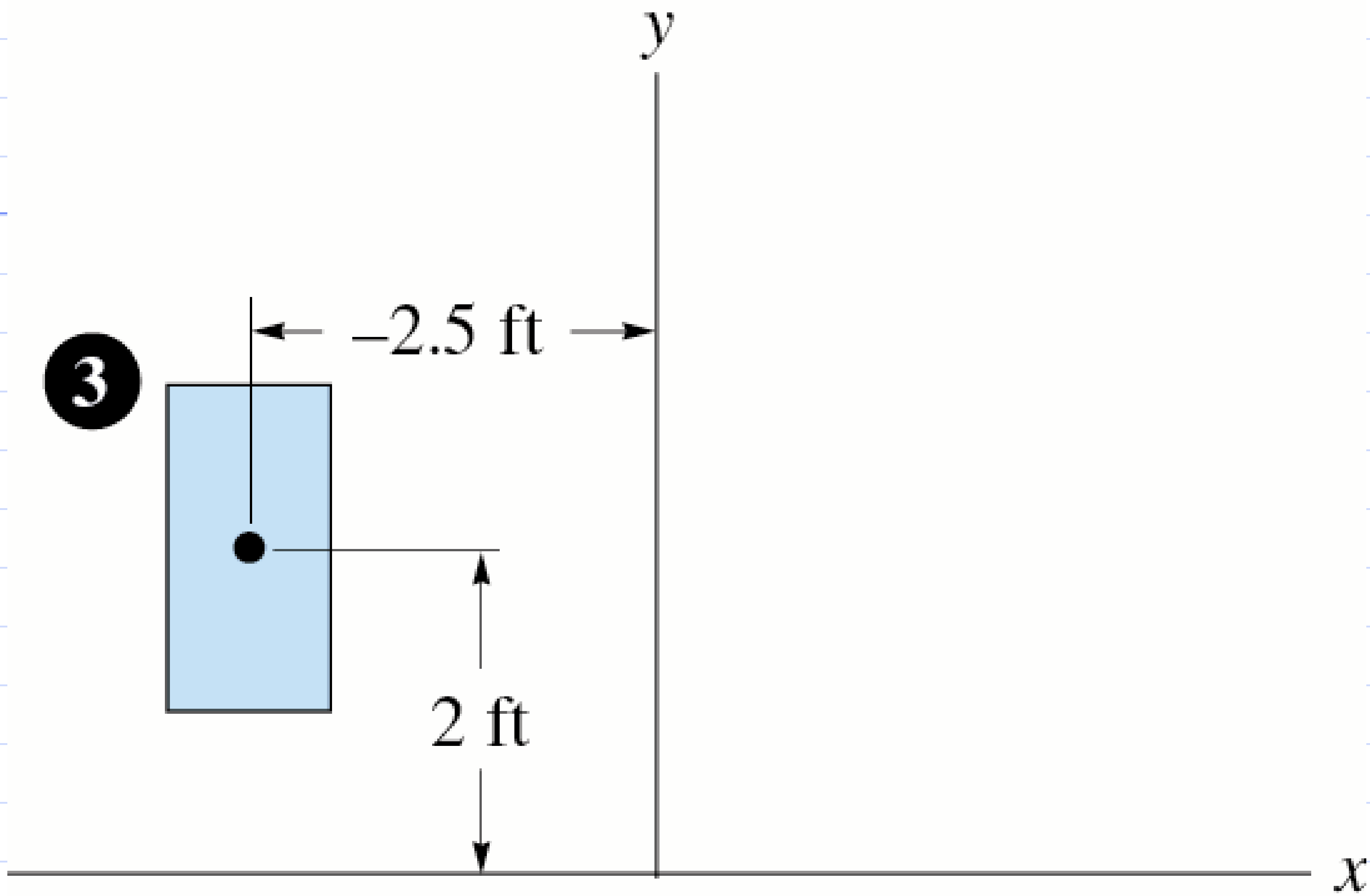
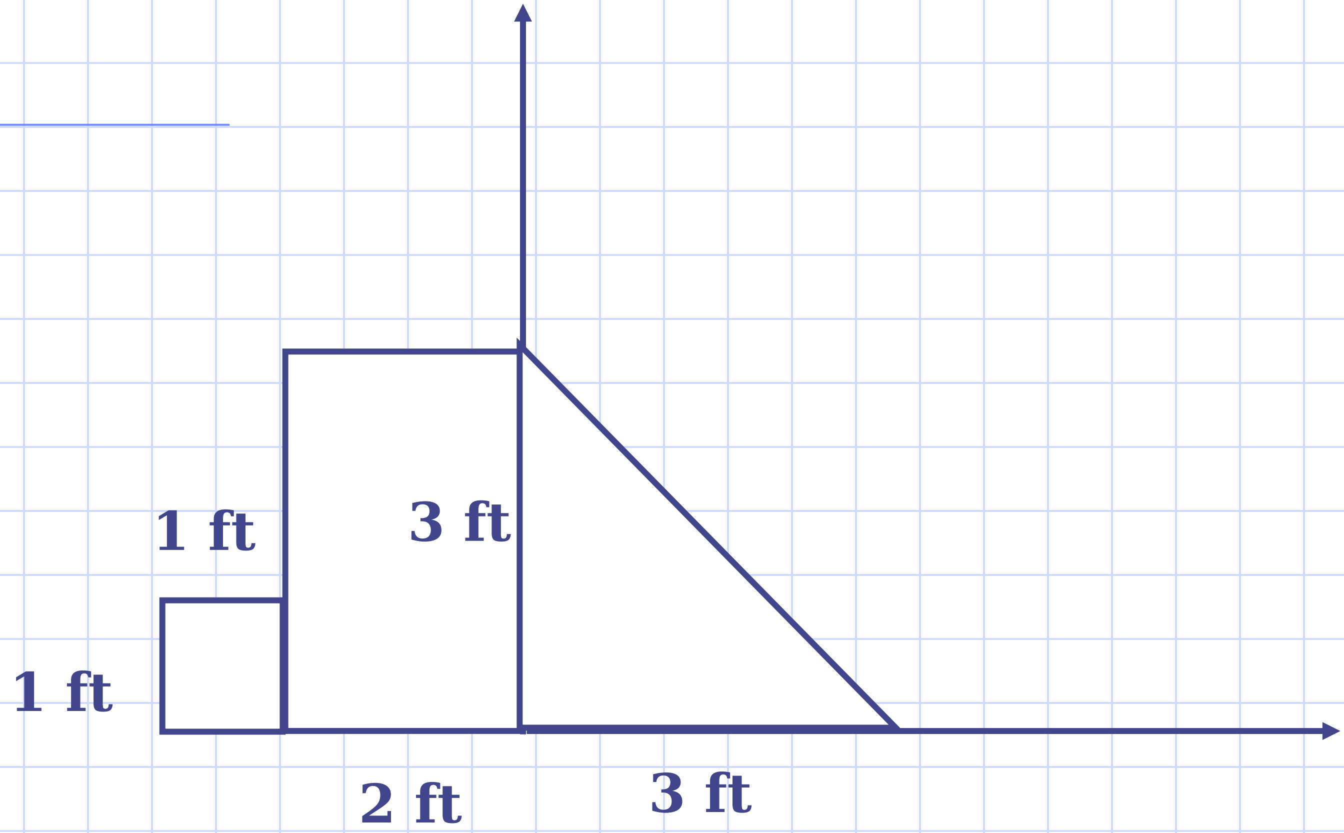
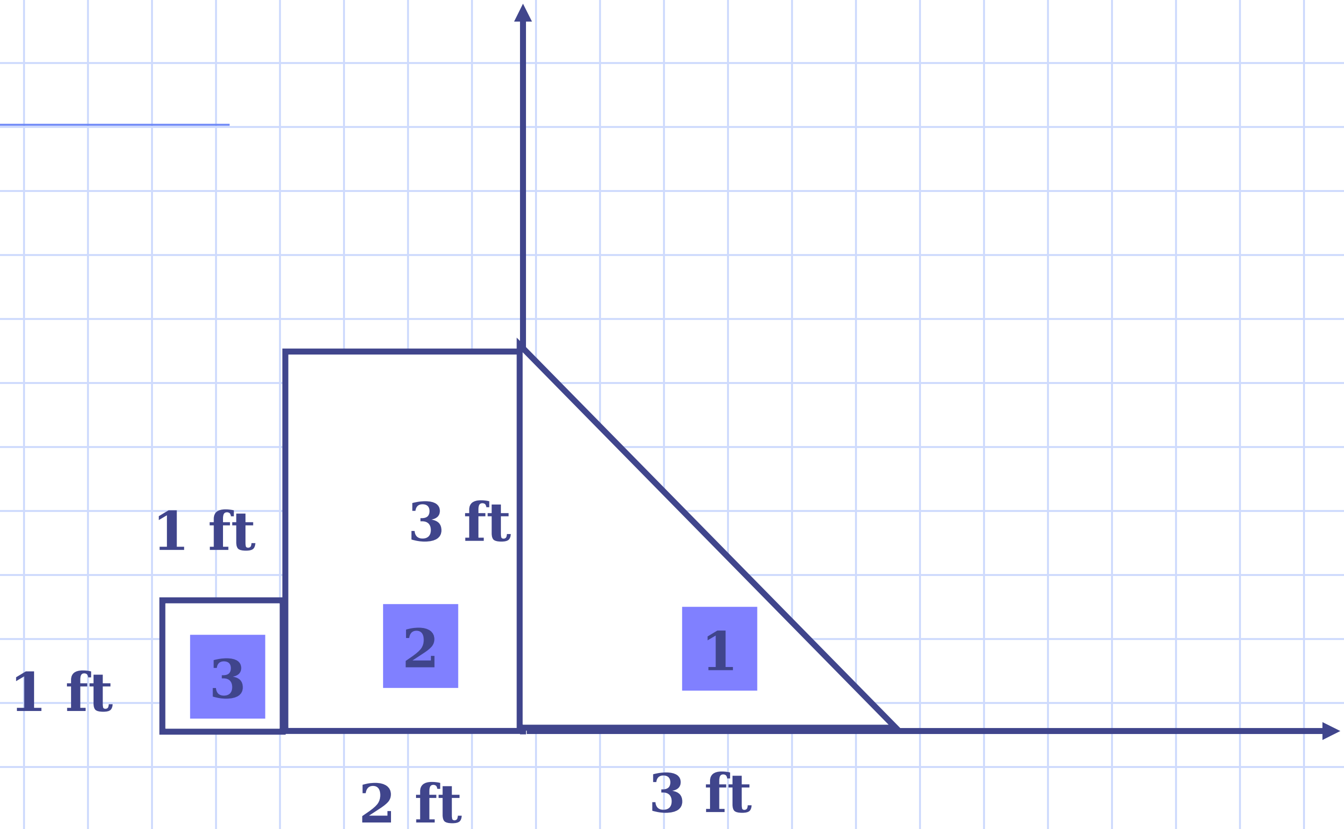


Figure 09.18(b2)



Locate Centroid of the Composite Area

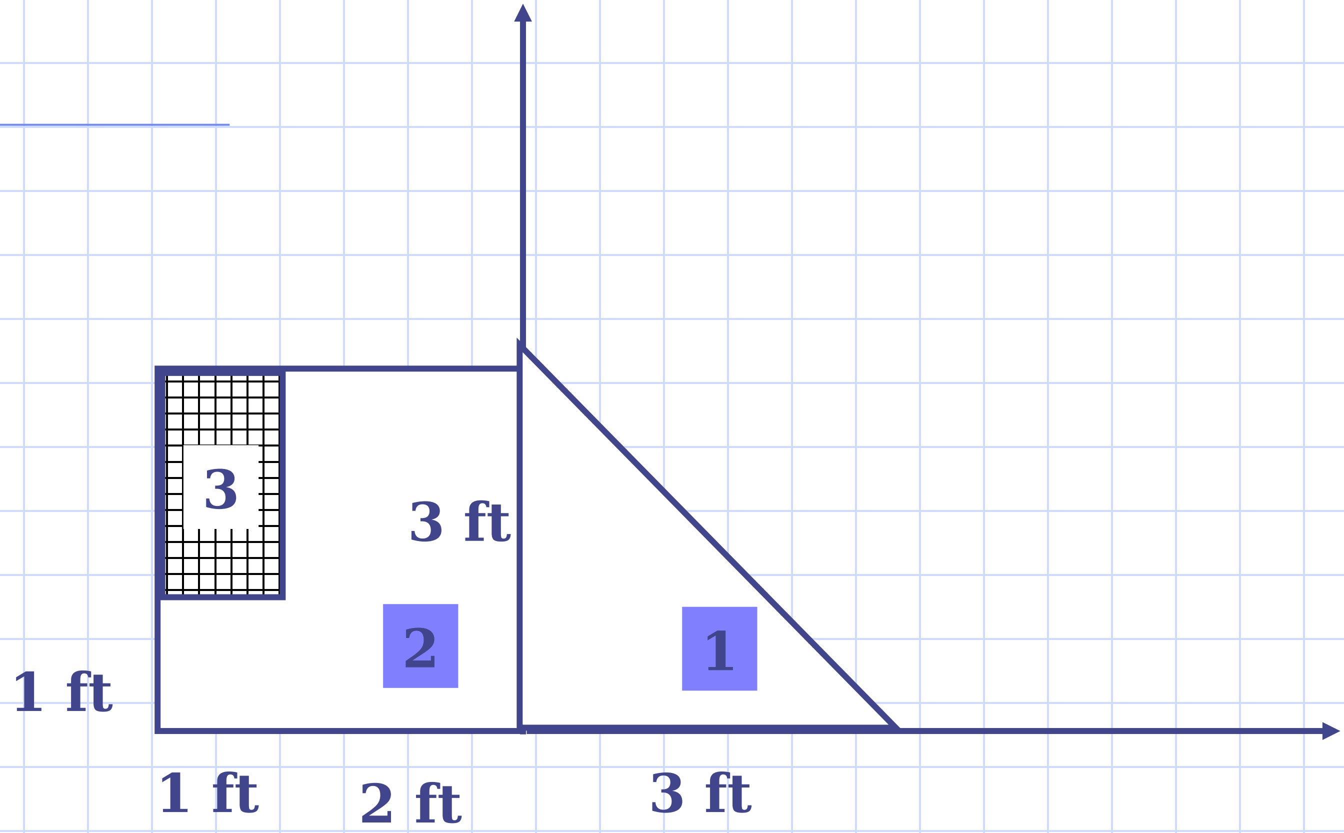

1 ft
3 ft
3
2
1
1 ft
2 ft
3 ft

Segment	A (ft²)	x	y	xA	yA
1	4.5	1	1	4.5	4.5
2	6	-1	1.5	-6	9
3	1	-2.5	0.5	-2.5	0.5

Σ A = 11.5 Σ xA = -4 Σ xA = 14

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} = \frac{-4}{11.5} = -0.348\ \text{ft}$$

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} = \frac{14}{11.5} = 1.22\ \text{ft}$$


3
3 ft
2
1
1 ft
1 ft
2 ft
3 ft

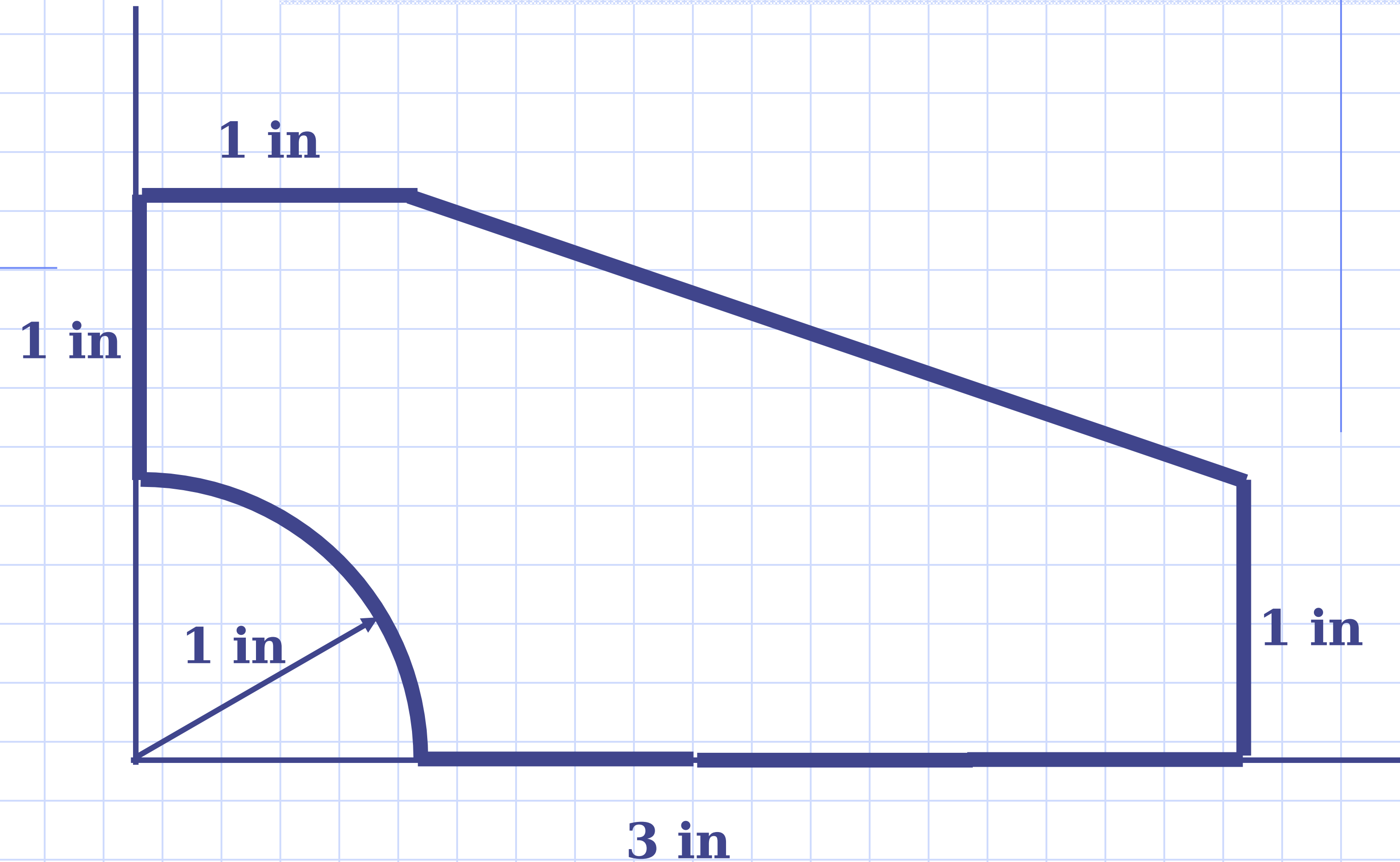
Segement	A (ft²)	x	y	xA	yA
1	4.5	1	1	4.5	4.5
2	9	-1.5	1.5	-13.5	13.5
3	-2.5	-2.5	2	5	-4

Σ A = 11.5   Σ xA = -4   Σ xA = 14

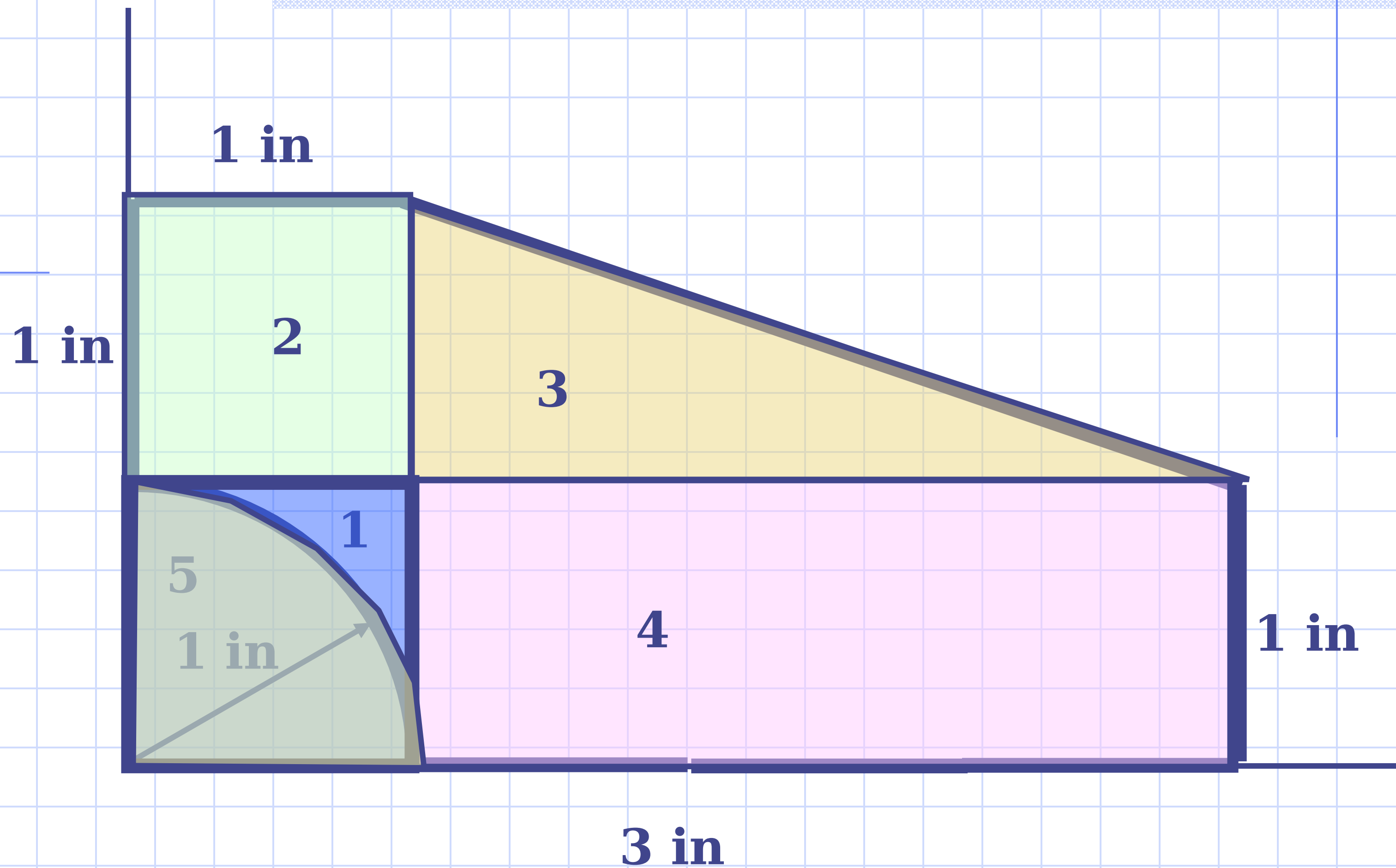
$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} = \frac{-4}{11.5} = -0.348\,ft$$

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} = \frac{14}{11.5} = 1.22\,ft$$

9.55



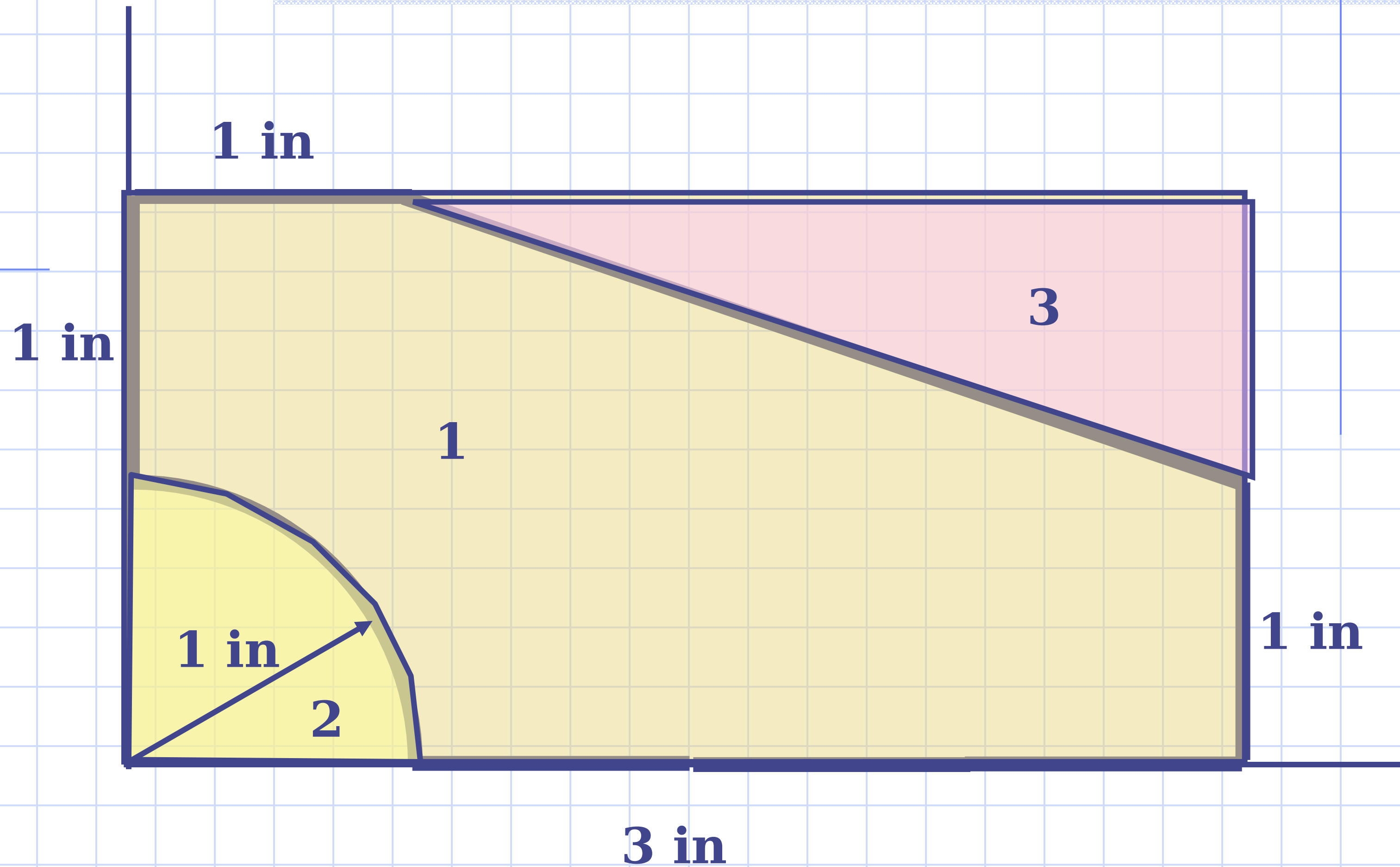
9.55



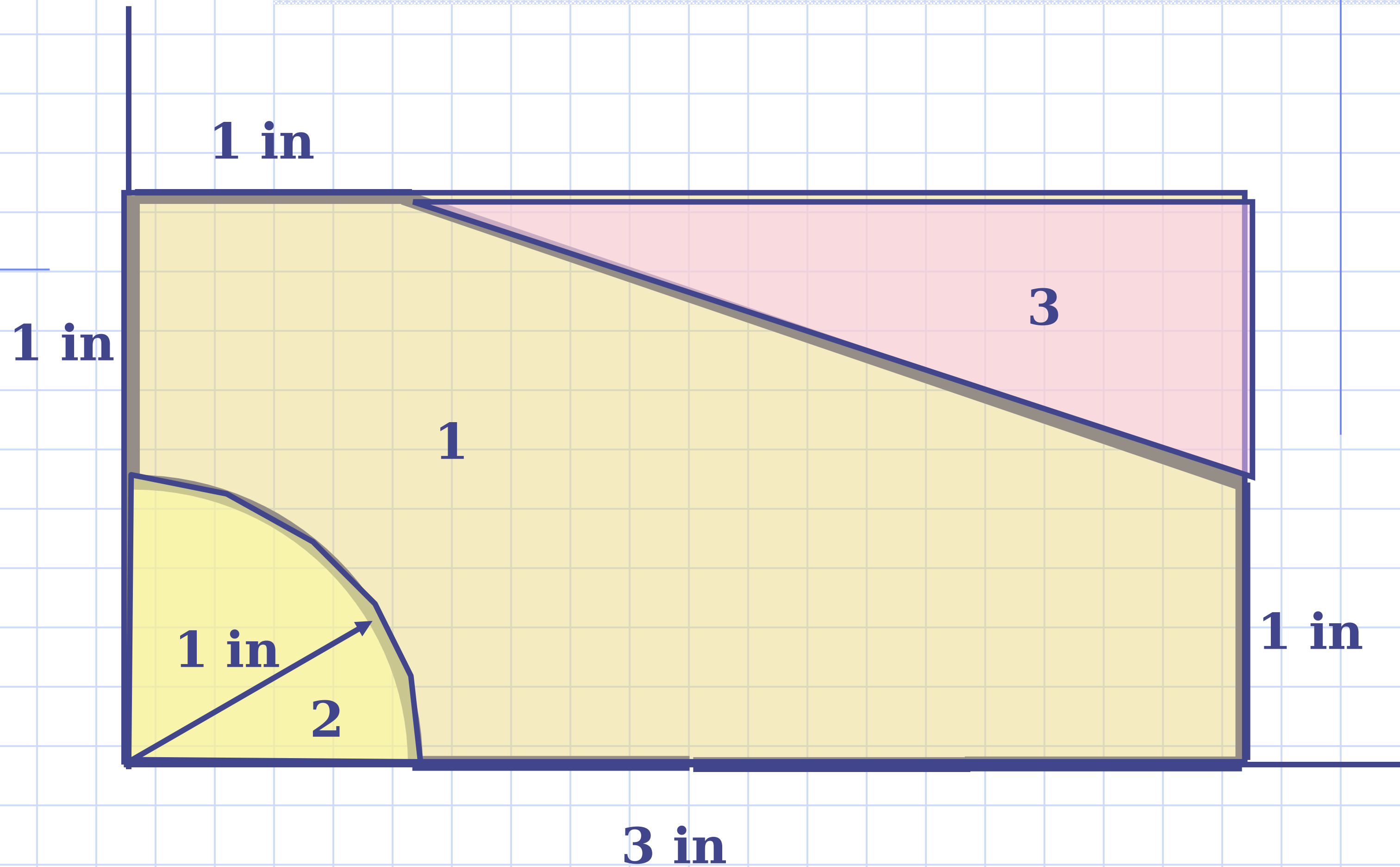
Break into sub-areas

Segment	Area	x	y	xA	yA
1.00000	1.00000	0.50000	0.50000	0.50000	0.50000
2.00000	1.00000	0.50000	1.50000	0.50000	1.50000
3.00000	1.50000	2.00000	1.33333	3.00000	2.00000
4.00000	3.00000	2.50000	0.50000	7.50000	1.50000
5.00000	-0.78540	0.42441	0.42441	-0.33333	-0.33333
	5.71460			11.16667	5.16667
	x=	1.95406			
	y=	0.90412			

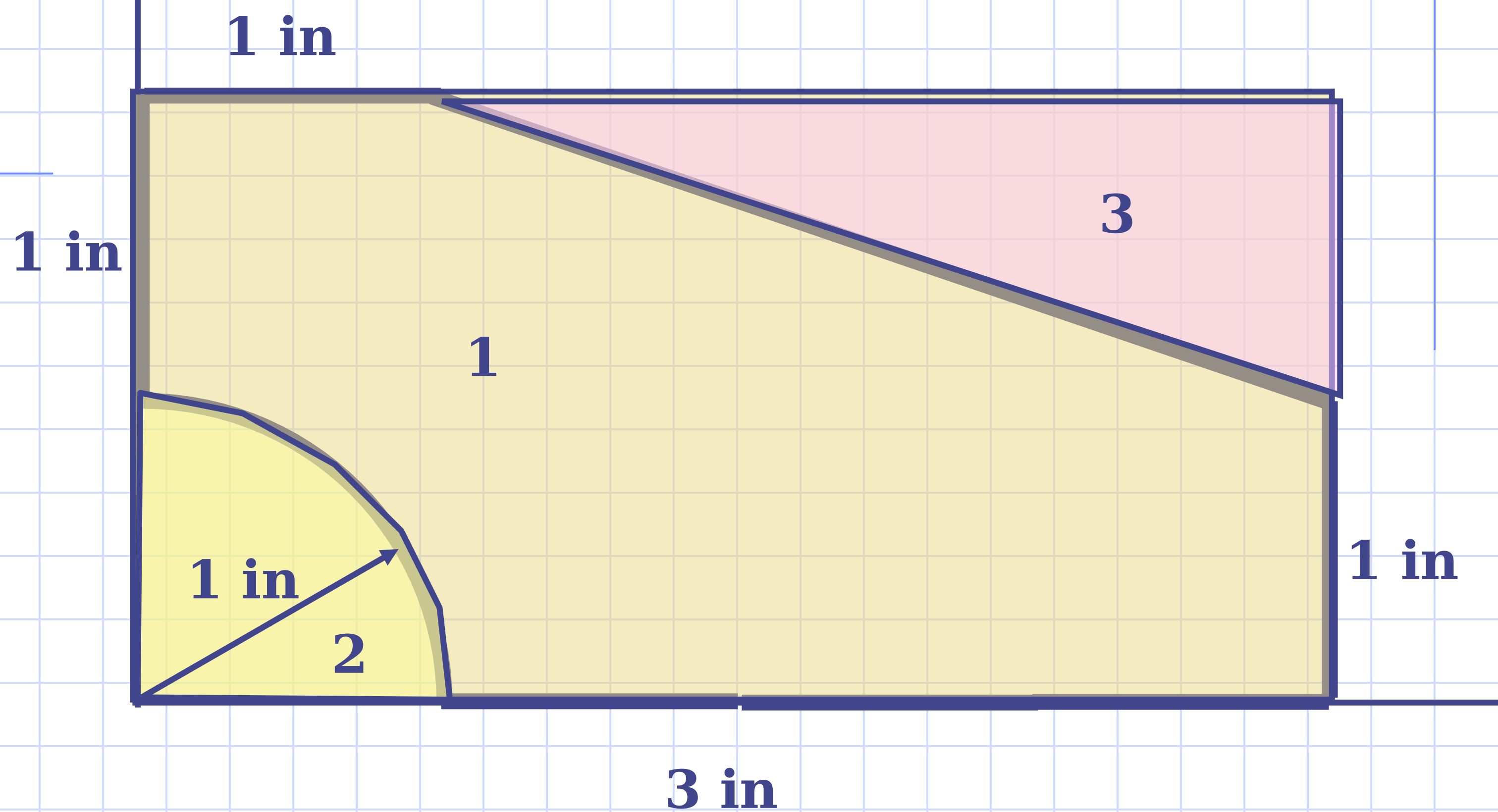
9.55



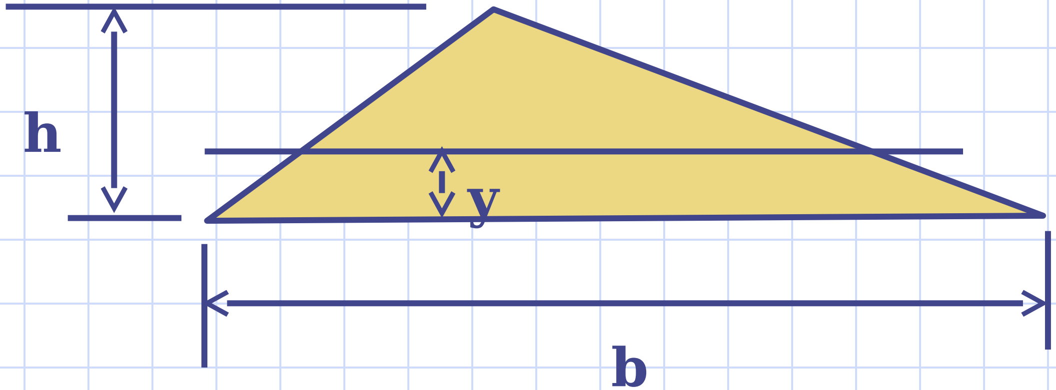
9.55



9.55

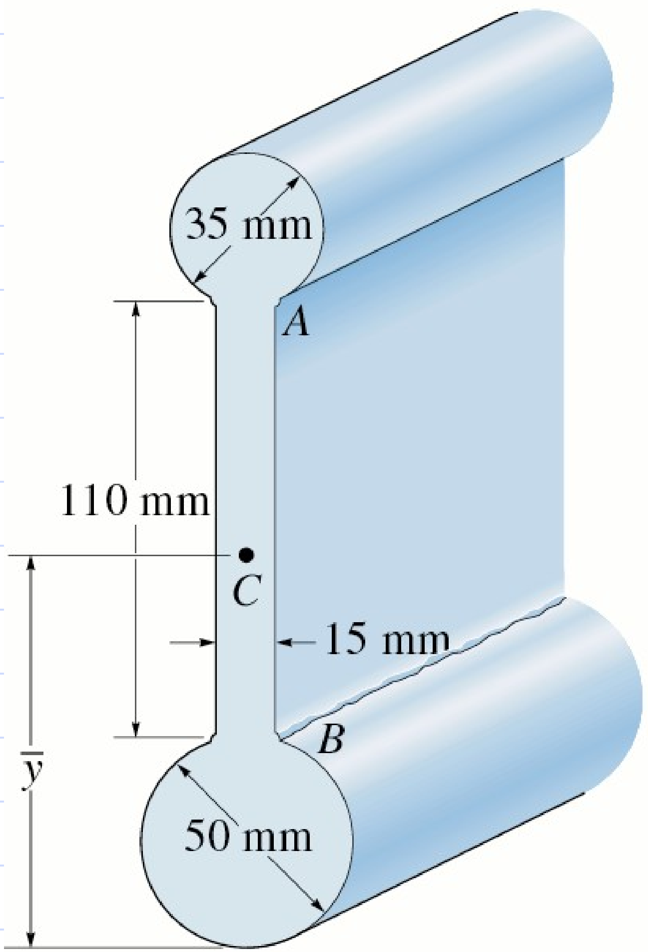


Segment	Area	x	y	xA	yA
1	8	2	1	16	8
2	-0.7854	0.424413	0.424413	-0.33333	-0.33333
3	-1.5	3	1.666667	-4.5	-2.5
	5.714602			11.16667	5.166667
	x=	1.954059			
	y=	0.904117			

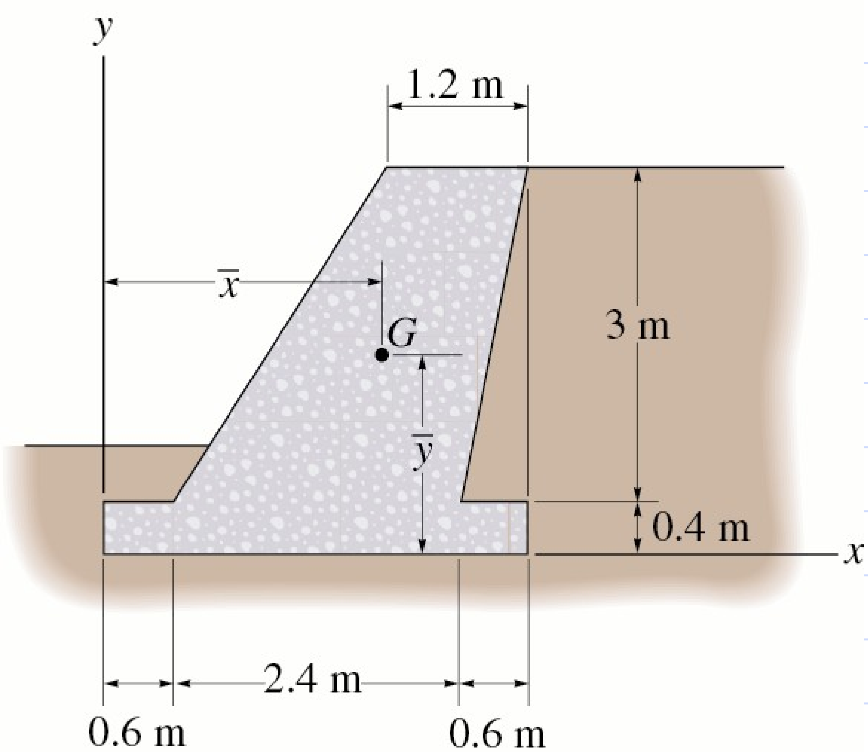


$$A = \frac{1}{2}bh$$

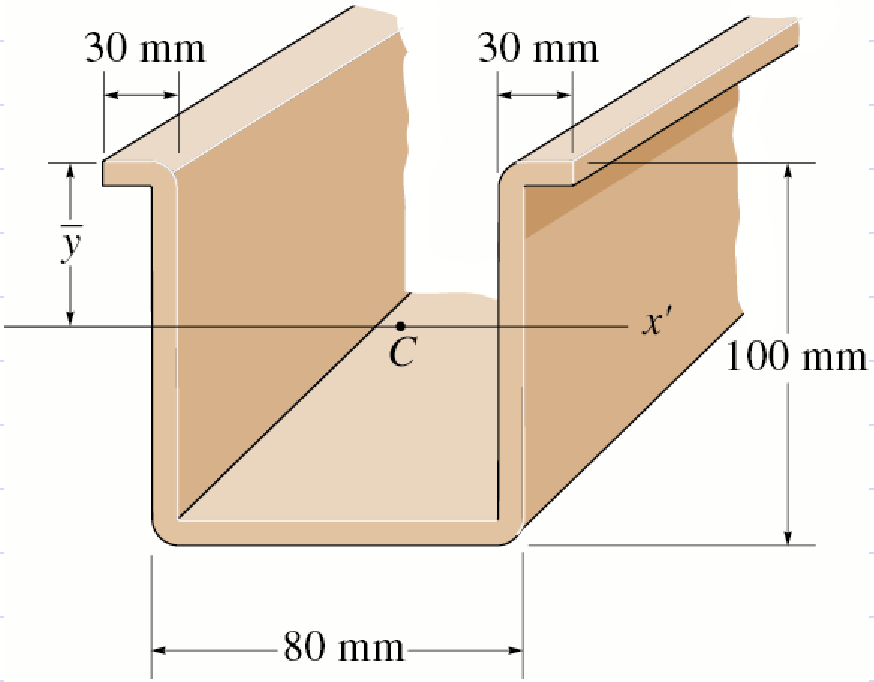
$$\bar{y} = \frac{h}{3}$$



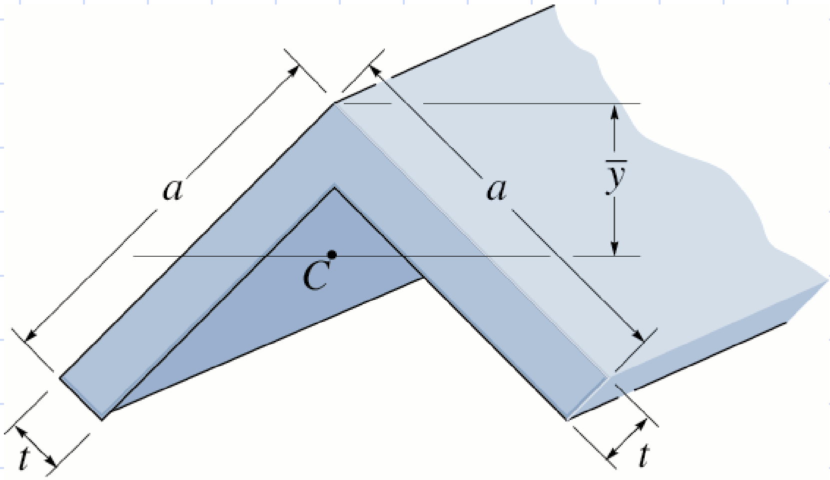
Prob 09.53



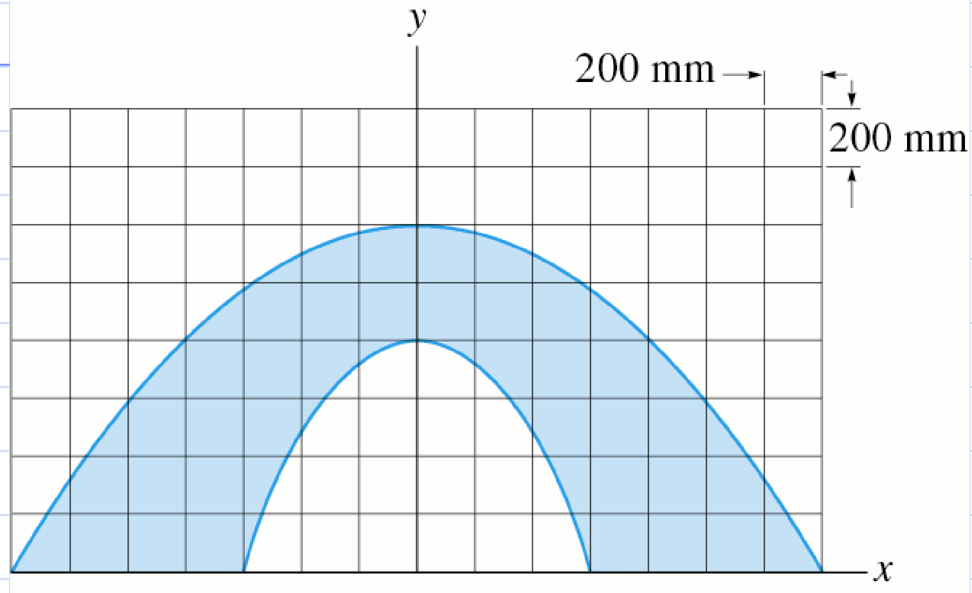
Prob 09.54



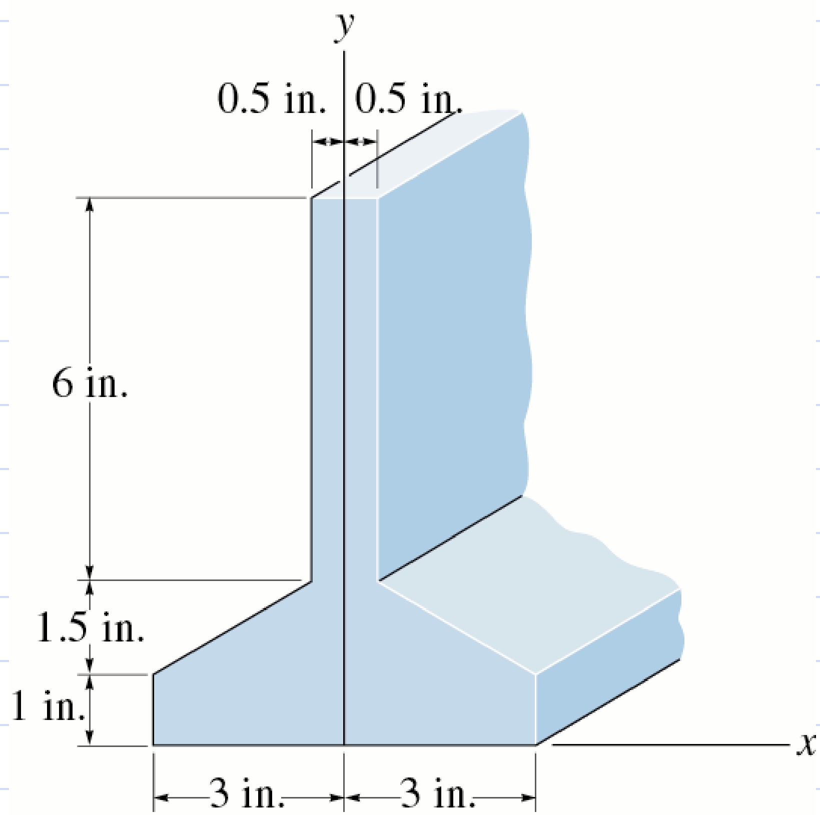
Prob 09.55



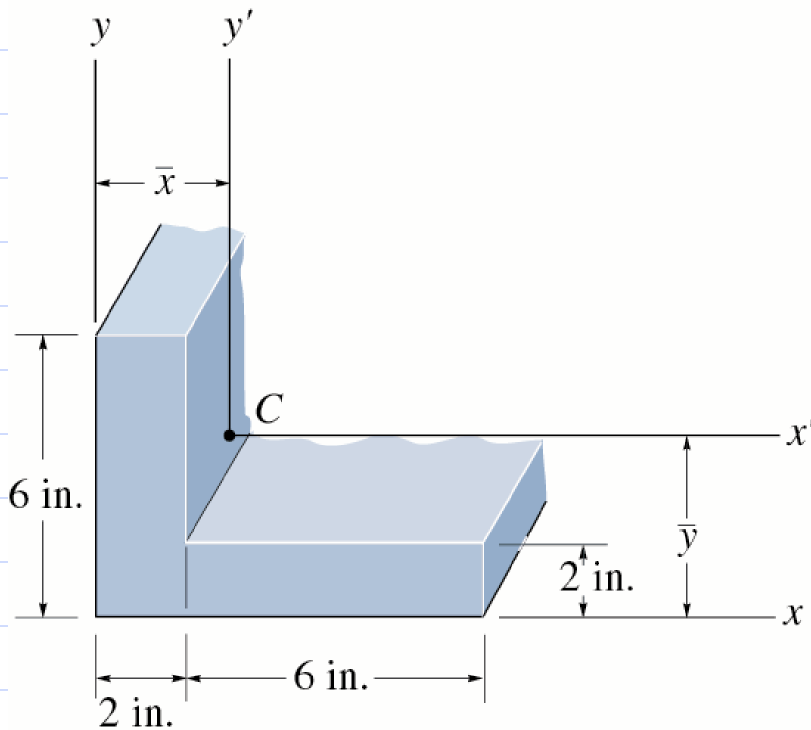
Prob 09.56



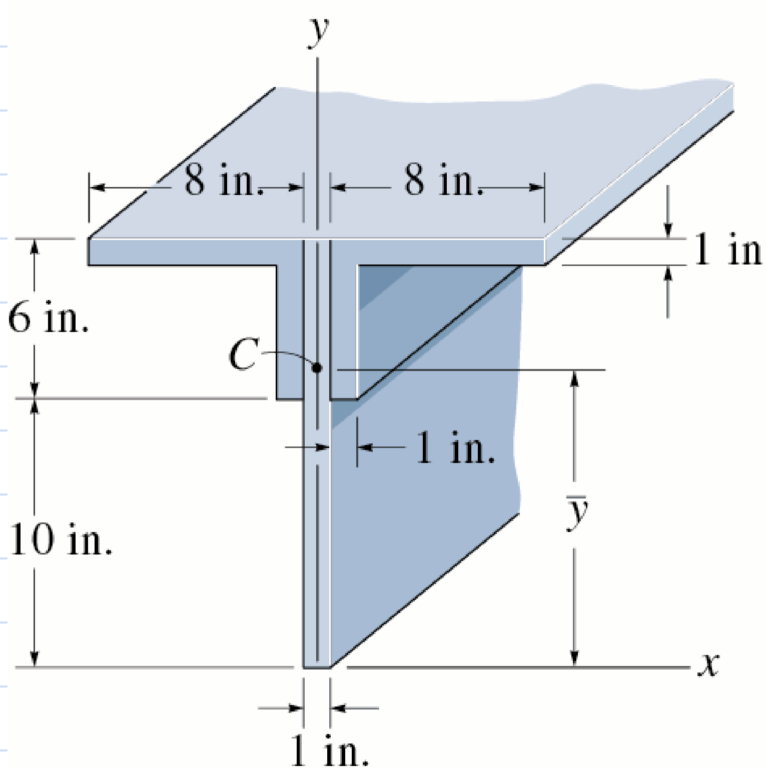
Prob 09.57



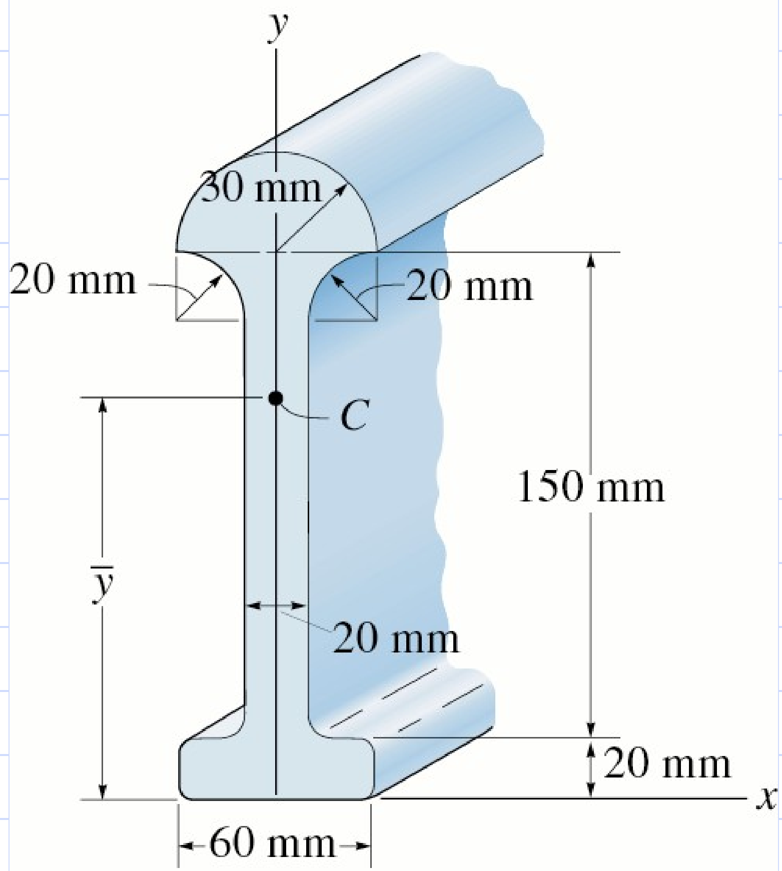
Prob 09.58



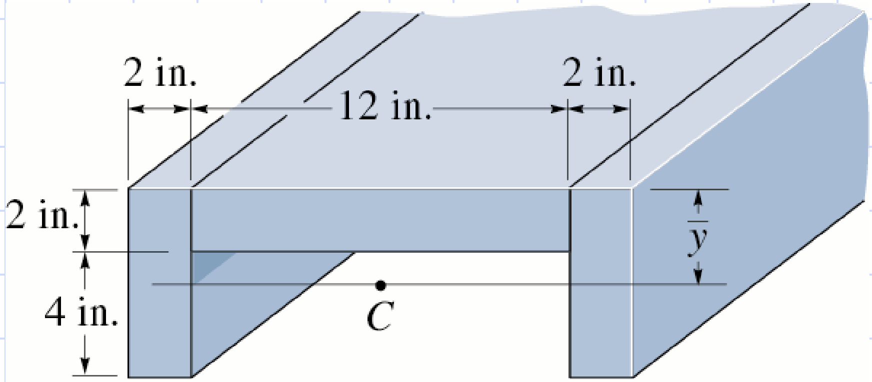
Prob 09.59



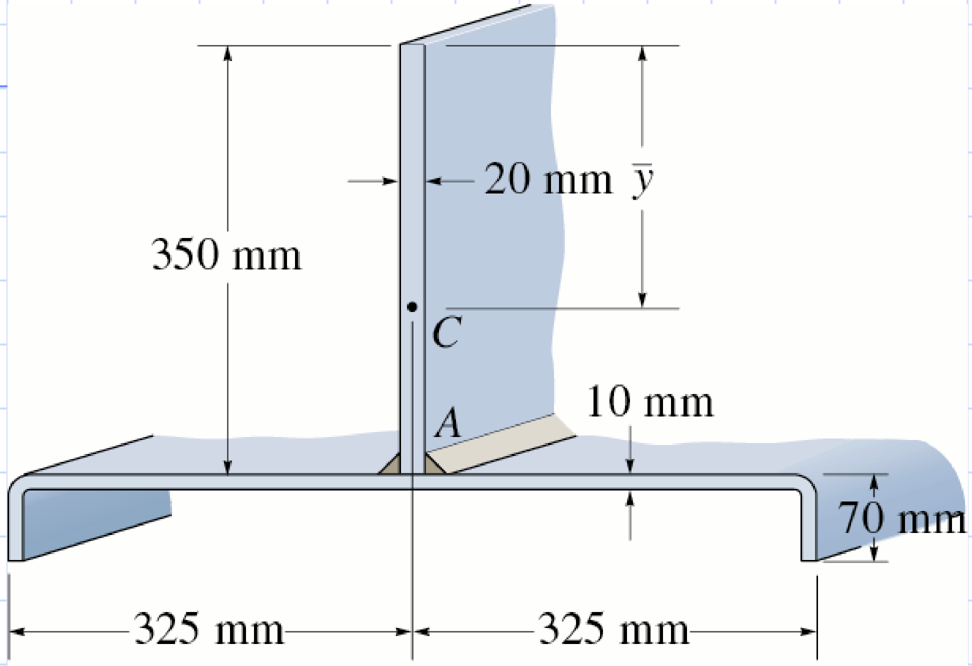
Prob 09.61



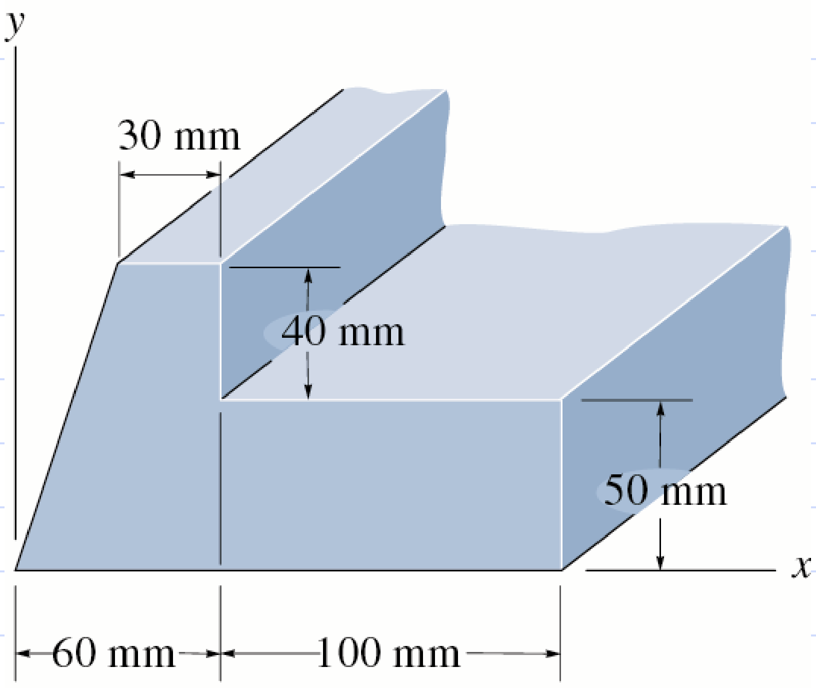
Prob 09.62



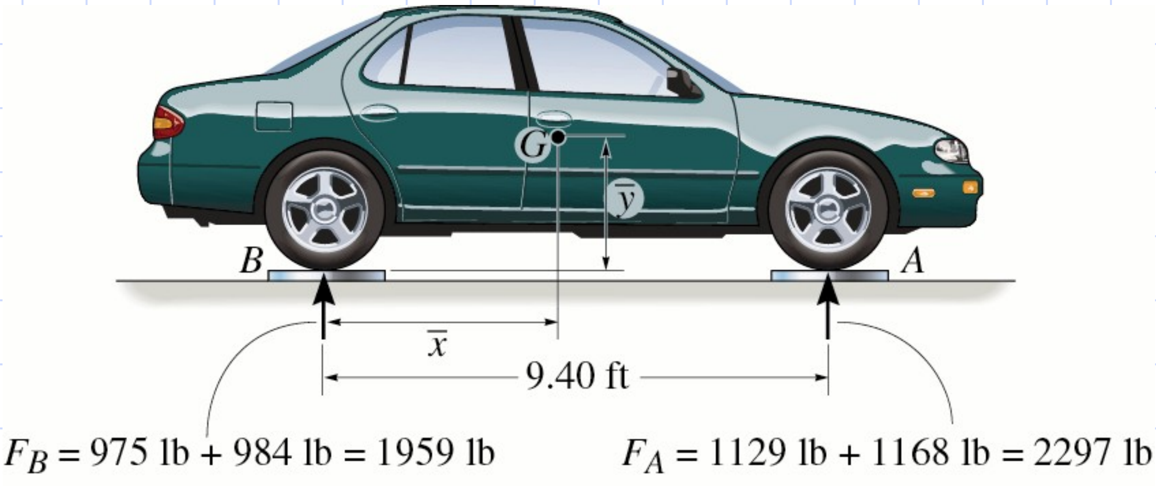
Prob 09.63

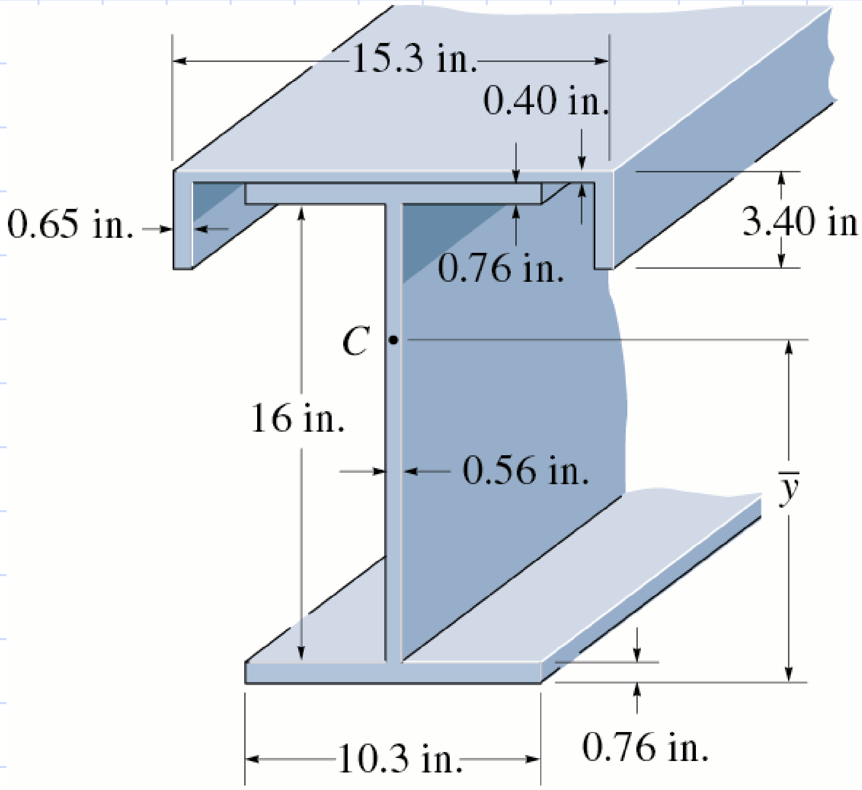


Prob 09.64

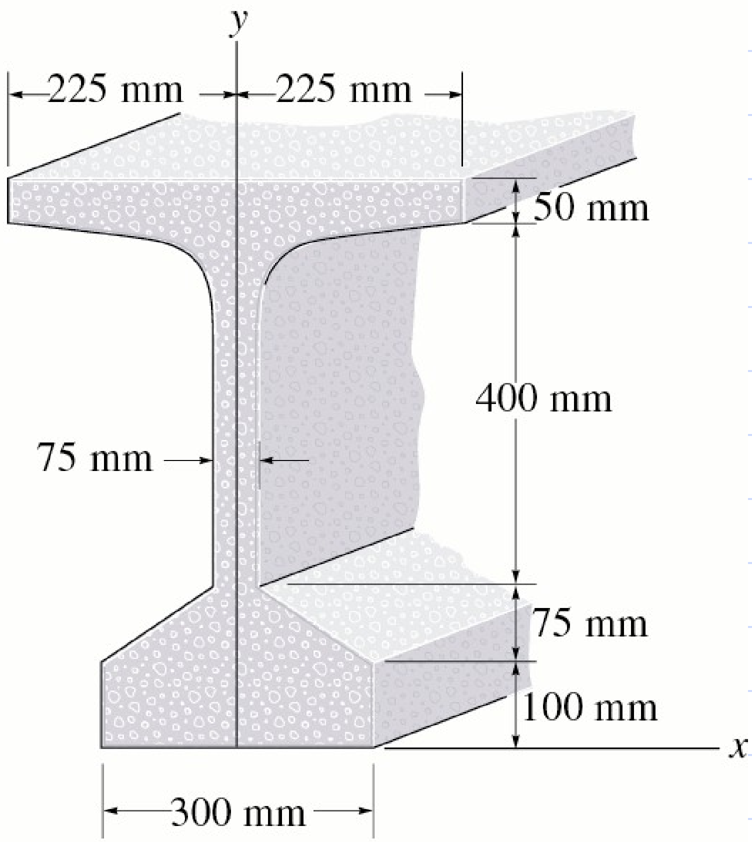


Prob 09.65

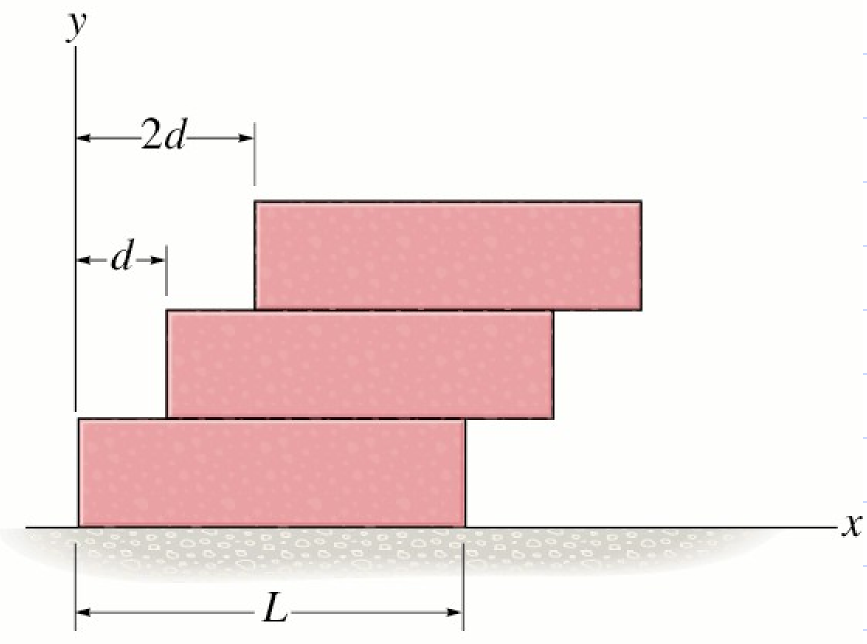

G
$\bar{y}$
B
A
$\bar{x}$
9.40 ft
$F_B = 975$ lb + 984 lb = 1959 lb
$F_A = 1129$ lb + 1168 lb = 2297 lb


15.3 in.
0.40 in.
0.65 in.
3.40 in.
0.76 in.
C
16 in.
0.56 in.
$\overline{y}$
10.3 in.
0.76 in.

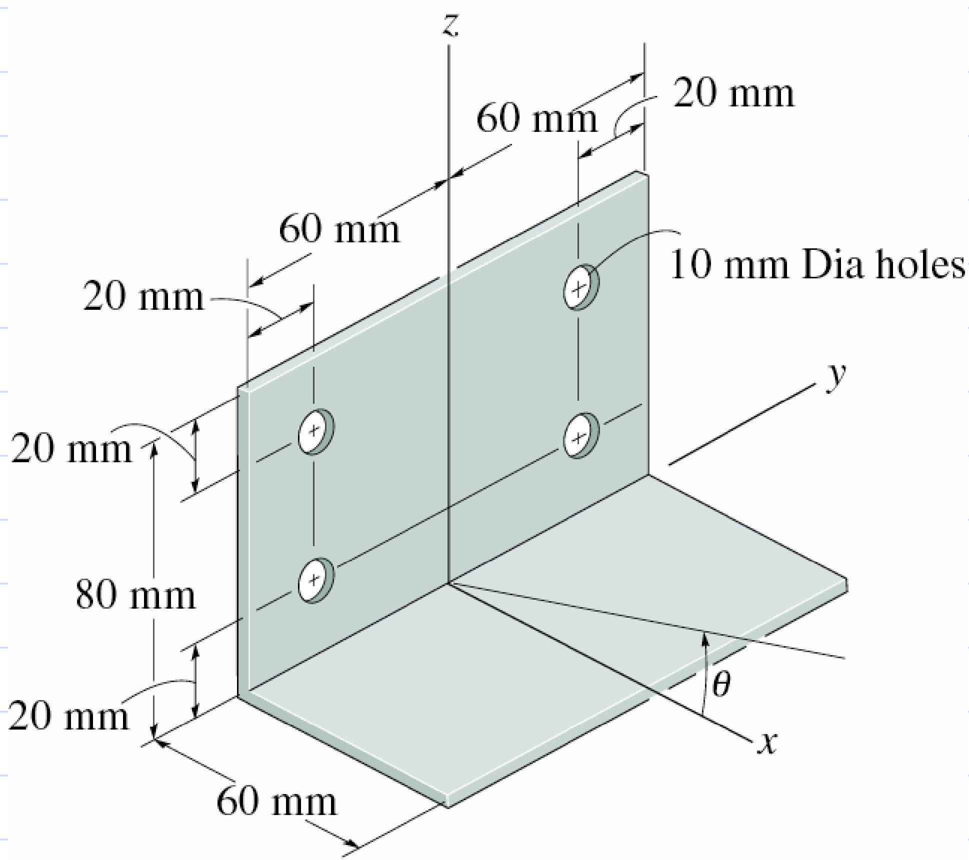
Prob 09.67



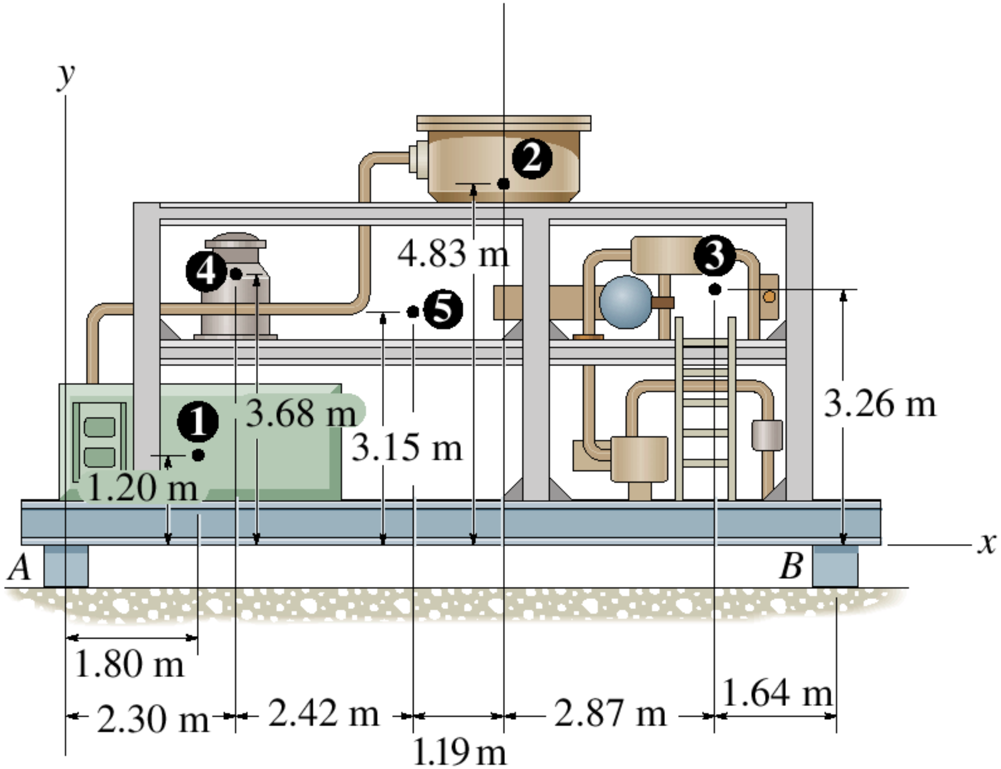
Prob 09.68



Probs 09.69/70



Prob 09.73



1	Instrument panel	230 kg
2	Filter system	183 kg
3	Piping assembly	120 kg
4	Liquid storage	85 kg
5	Structural framework	468 kg

Prob 09.74